

The non-linear wave theory, adequate of Standard Model

Alexander G. Kyriakos

St. Petersburg, Russia.

*E-mail: a.g.kyriak@hotmail.com
agkyriak@hol.gr*

Abstract

A solitary stable wave – soliton - is defined as a spatially confined (localized), non-dispersive and non-singular solution of a non-linear wave theory. For any non-linear wave theory the solitons are the same fundamental solutions, as the usual waves are the fundamental solutions of the linear equations. For elementary particle physics a localized and stable wave is a perfect model for elementary particles, opening up in a non-linear field theory the possibility of what would have to be a wave packet in a linear one. As it is known the newer fundamental non-Abelian gauge theories are non-linear and have the soliton solutions. In the framework of the quantum field theory it is not difficult to find the relations between solitons and elementary particles that go very deep and are entirely unexpected from the classical point of view.

In the present paper is offered the theory of the waves with new type of non-linearity, emergent thanks to the transformations of the gauge type. The peculiar solitons are the constituents of this theory, which are identical with the objects of Standard Model. In particular they have masses, which appear due to the spontaneous breakdown of symmetry of the initial waves; they can be only in two states – bosonic and fermionic; they can have positive and negative charges, etc, etc.

The mathematical description of the present theory can be interpreted within the framework of the Copenhagen tradition, but, as it is shown below, non-linear theory interpretation is much more consecutive and productive.

A table of contents

Introduction to the theory	(1)
Chapter 1. The theory axiomatics	(5)
Chapter 2. The electron theory	(7)
Chapter 3. Quantum and electromagnetic forms of electron theory	(20)
Chapter 4. Point and non-point solutions of electron equations	(29)
Chapter 5. The massive neutrino theory	(36)
Chapter 6. On the hadrons theory	(46)
Chapter 7. On elementary particles' spectra	(53)
Bibliography	(64)

Introduction to the theory

The Standard Model theory (SM) is the gauge theory or the theory of massless vector fields, in which the particles acquire the masses thanks to the spontaneous breakdown of the physical vacuum symmetry. To understand the connection of the present theory with SM, we examine one simple example, which will be described in detail and generalized in following chapters of this book.

As one of the examples of the creation of massive field (i.e. particles) from the massless vector field it can serve the case of the of electron-positron pair photoproduction:

$$\gamma + N \rightarrow e^+ + e^- + N , \quad (1)$$

Actually, the gamma-quantum (for simplicity sake, photon) γ is the massless boson, whose field is converted as vector. The nucleus field N initiates transformation of this photon into two massive particles e^+, e^- - electron and positron, whose fields are spinors, which are converted differently in comparison with the vector field. Thus, it can say that equation (1) describes the process of the symmetry breaking of initial massless vector field with the result - the creation of massive spinor particles. Simultaneously, equation (1) shows that the transition from boson to spinor field is connected with some specific transformation. To find the last one we consider this process as the Feynman diagram (fig. 1):

Fig.1

We know nothing about what happened in the “birthpoint” of the pair. We only see here the beginning and the end of the process.

But what transformation could take place with massless photon in a field of an atom nucleus, which has led to the appearance of two, relatively motionless particles, both with mass and spin, which are equal to half of energy and spin of a photon, and also with mutually opposite electric charges? Let’s try to answer this question.

The real photon structure is unknown to us. But a photon is the quantum of electromagnetic (EM) wave and therefore the bending of a trajectory of EM wave in the strong EM field must have place in this case.

So, we can suggest that under above conditions the photon EM wave, as some string, can start to move along the closed curvilinear trajectory, forming the objects, which as a single whole must have mass and therefore cannot move with the light speed.

Below we show that such objects, depending on the kind of trajectory, will have all the characteristics and parameters of real elementary particles and have described by known relativistic equations of Dirac and Yang-Mills (the details see in the mentioned chapters).

According to above suggestion in simplest case of Fig. 1 a photon, as EM wave string, should to be twirled into a ring, and then it should be divided into other two rings, which can move now with a speed less than the speed of light. Thus, a “linear” photon - i.e. photon obeys the linear wave equation - must became the non-linear photon and then produces two non-linear waves, which must obey other equation.

Obviously, a twirled photon will have the mass that is equal to energy of a “linear” photon, divided on a square of light speed, and, as it is easy to show, the spin, equal to one. Apparently, after photon division we receive two particles with rest mass equal to half of mass of a twirled photon and with spin equal to half of the spin of a photon. Let’s try to find the basis of theoretic description of this process (see in detail the chapter 2).

Let us return to Fig.1 and recall the description of the corresponding particles. As we know, photon is described by means of the quantized equation of Maxwell. The electron and positron are described by the equations of Dirac. Thus, conditionally speaking, we see how to the left the equation of Maxwell of the quantized electromagnetic waves “flies” into the strong electromagnetic nuclear field of atom. Then we see how to the right two equations of Dirac (one for the electron, and another for the positron) “depart”.

Thus, according to our scheme it follows that the Dirac equations are the EM field equations of two parts of the twirled quantized EM wave. We can say also that in the intersection point of the Feynman diagram lines a rotation transformation is realized. We can assume that this transformation is identical to the gauge transformation because the last one is (see e.g. (Ryder, 1985)) the transformation of field rotation in the inner symmetry space of particle.

Does this assumption contradict to the modern quantum field theory?

At first we know that (Gspomer, 2002) “the conventional view is that spin 1 and spin $\frac{1}{2}$ particles belong to distinct irreducible representations of the Poincare group, so that there should be no connection between the Maxwell and Dirac equations describing the dynamics of these particles”. How this is in agreement with the fact that the offered elementary particle theory is electromagnetic?

It is of course true that the Dirac equation is not equivalent to classical Maxwell equations (although for a long time it is established that they can be formally presented in the identical mathematical form). But in the present theory there is not a question about Maxwell theory, but about the special non-linear electromagnetic wave theory. The wave function of the Dirac equation appears here as a result of transformation and breaking of photon vector wave function to the spinor wave functions in fully accordance with equation (1).

Let’s remember in this connection, that the Dirac equation in the fiftieth years of the previous century was named the “semi-vector” equation, and their wave function was named the “semi-vector” (see, for example, (Goenner, 2004)), because the last one is connected with the vector field by certain mathematical relations (see e.g. (Ryder, 1985)).

Secondly, it seems there is a difficulty: the Maxwell time depending equations contain six vectors and six equations (the source equations are possible to consider as the initial conditions); at the same

$$\text{rot}\vec{H} = \frac{4\pi}{c} (\vec{j} + \vec{j}_{ext}) + \frac{1}{c} \frac{\partial \vec{E}}{\partial t}, \quad (1.1)$$

$$\text{rot}\vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \quad (1.2)$$

$$\text{div}\vec{E} = 4\pi(\rho + \rho_{ext}), \quad (1.3)$$

$$\text{div}\vec{H} = 0, \quad (1.4)$$

where $\vec{F} = \{\vec{E}, \vec{H}\}$ is the electromagnetic field vectors, ρ is the charge density; \vec{j} is the current density; c is the speed of light. Values \vec{j} and ρ (or in 4-vector form $j_\mu = \{\vec{j}, i\rho\}$, where $\mu = 1, 2, 3, 4$) in these equations should be considered as functions (more precisely, functionals) of strength \vec{E} and \vec{H} of the same fields, which these charges and currents substantially define: $j_\mu = j_\mu(\vec{E}, \vec{H})$. The part of charges and currents can be caused by external in relation to the given system reasons. Such charges and currents, which do not dependent on \vec{E} and \vec{H} of an initial source, sometimes are referred to as external and are designated as \vec{j}_{ext} and ρ_{ext} . (We shall remind, that generally the fields and currents of these equations are interdependent, and the equations are thereof non-linear).

As is known, the Maxwell-Lorentz theory predict the existence of electromagnetic waves. In relation to these the following postulate has place.

2. Plank's-Einstein's postulate of quantization of electromagnetic waves: *the electromagnetic waves are the superposition of the elementary wave fields named photons, having the certain energy, momentum and zero rest mass.*

In this postulate are simultaneously taken into account both quantization of electromagnetic waves and the belonging of photons to bosons.

3. A postulate of dualism of photons: *photons exist as real independent objects, which have*

a) the wave properties, described by Maxwell-Lorentz equations (1.1) - (1.4) and by the wave equation following from them:

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \vec{F} = 0, \quad (1.5)$$

b) quantum properties, which are not deduced from Maxwell-Lorentz theory, but also not contradicted to it.

The numerical characteristics of photons are determined by a below postulate of Planck.

4. Planck postulate: *Connection between energy, frequency and wavelength of a photon is set by the following formulas:*

$$\varepsilon = h\nu = \hbar\omega, \quad (1.6)$$

$$\lambda = \frac{h}{p} = \frac{hc}{\varepsilon}, \quad (1.7)$$

where ε , ν , ω , λ , p , c are the energy, linear frequency, circular frequency, wavelength, momentum and speed of a photon, and h , \hbar are the usual and bar Planck's constants accordingly.

The following postulate is neither proved nor disproved by experiments, but it doesn't contradict to the theoretical description of photons:

5. The postulate of EM string.

Since this postulate is central in our theory, it demands a serious substantiation of its consistency to modern results.

As it is known, in framework of QED (Ahiezer and Berestetski, 1969), to obtain the photon wave function the second order wave equations for EM field vectors \vec{E} and \vec{H} (1.5) are used.

Factorizing the wave equation to the equations for retarded and advanced waves, we receive two equations of the first degree regarding the function f_k , which adequate to a wave vector k and is some

generalization of the EM field vectors. The equation for this function is equivalent to the Maxwell-Lorentz equations.

But the function f_k can be interpreted as wave function of a photon only in the *momentum space*. It does not allow to describe an interaction of a photon in the local point of *coordinate space*, since for this aim the wave function in the coordinate representation is required.

Unfortunately the attempt to enter the photon function in the coordinate representation has strike on an insuperable difficulty. According to analysis of Landau, L.D. and Peierls, R. (Landau and Peierls, 1930) and later of Cook, R.J. (Cook, 1982a;1982b) and Inagaki, T. (Inagaki, 1994) the photon wave function is nonlocal.

Actually, having made the inverse Fourier transformation of above f_k function $\frac{1}{(2\pi)^3} \int f_k e^{i\vec{k}\vec{r}} d^3k = f(\vec{r}, t)$, it is possible to define $f(\vec{r}, t)$ as the photon wave function in coordinate representation. But the $f(\vec{r}, t)$ function is not defined by the value of the field $\vec{E}(\vec{r}, t)$ in the same point; it depends on the field distribution in some area, which sizes are of the order of the photon wavelength. This means, that the localization of a photon in a smaller area is impossible and, hence, the value $|f(\vec{r}, t)|^2$ will not have the sense of probability density to find a photon in the given point of coordinate space.

The linear object, which, on the one hand, obeys the wave equation, and on the other hand has some size, is referred to as a *string*. Thus, within the framework of CWED it is admissible to describe a photon as an element of electromagnetic wave - an electromagnetic string (not forgetting of course that this supposition cannot have any relationship to the real structure of a photon). This allows us to formulate the following postulate:

Within the framework of the present theory the fundamental particle of an EM field - the photon - can be described as a relativistic EM string of one wavelength size, which corresponds to its energy according to Planck's formula.

The main proof of validity of this postulate is the opportunity to construct on its basis the theory, which coincides completely with the existing quantum field theory.

6. A postulate of formation of massive particles: *Within the framework of the present theory under the certain external conditions the EM-string can start to move along the closed curvilinear trajectory, forming the elementary particles.*

As is known, the bending of a trajectory of an EM wave in the strong EM field follows already from the Maxwell-Lorentz theory. Thus, strictly speaking, the opportunity of an EM wave propagation along a curvilinear trajectory does not demand a special postulate.

At the same time, it is obvious, that due to the quantum nature of a photon (EM-string) the formed particles should possess, at least, a rest mass and the angular momentum (spin). Moreover, the detail analysis shows that such elementary particles can have electric charge, helicity and all other characteristics and parameters of real elementary particles.

It would be very difficult to find simultaneously the adequate description of all EM elementary particles. The most logical way of construction of the general theory - to begin with the most simple and good studied - theoretically and experimentally - particle: the electron. Chapters 2-4 of the book are devoted to this. Then we shall try to generalize the received results for the description of more exotic and complex particles.

Further for brevity this theory is referred to as **CWED** - Curvilinear Waves' Electrodynamics.

Chapter 2. The electron theory

1.0. Introduction

The possibility of the formal representations of the Schreudinger and the Dirac electron equations in the form of the linear Maxwell equations was mentioned in several articles and books (Archibald, 1955; Akhiezer and Berestetskii, 1965; Koga, 1975; Campolattoro, 1980; Rodrigues, 2002).

According to postulate 6, an electromagnetic wave, which move along the closed curvilinear trajectory, must create the stabile objects that correspond to elementary particles of different kind.

Let's now translate this supposition into the mathematics language and show that in the simplest case the matrix form of the equations of such curvilinear waves mathematically fully coincides with

quantum equations of vector and spinor (semi-vector) particles and gives many interesting consequences, which supplement the quantum field theory results.

2.0. Linear EM wave equation in the matrix form

We define as “linear” wave the solution of the linear wave equation.

Let us consider the plane-polarized linear electromagnetic (EM) wave moving, for example, on y - axis, the electric and magnetic fields of which can be written in the complex form as:

$$\begin{cases} \vec{E} = \vec{E}_o e^{-i(\omega t \pm ky)} \\ \vec{H} = \vec{H}_o e^{-i(\omega t \pm ky)} \end{cases}, \quad (2.1)$$

The electromagnetic wave of any direction has two plane polarizations and contains only four field vectors; for example, in the case of y -direction we have:

$$\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}, \quad (2.2)$$

and $E_y = H_y = 0$ for all transformations. Note in this connection that the Dirac bispinor has also four components.

The EM wave equation has the following known view (Jackson, 1999):

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) \vec{\Phi}(y) = 0, \quad (2.3)$$

where $\vec{\Phi}(y)$ is any of the above electromagnetic wave field vectors (2.2). In other words this equation represents four equations: one for each vectors of the electromagnetic field.

We can also write this equation in the following operator form:

$$\left(\hat{\varepsilon}^2 - c^2 \hat{p}^2 \right) \Phi(y) = 0, \quad (2.4)$$

where $\hat{\varepsilon} = i\hbar \frac{\partial}{\partial t}$, $\hat{p} = -i\hbar \nabla$ are the operators of the energy and momentum correspondingly and

Φ is some matrix, which consists four components of $\vec{\Phi}(y)$.

Taking into account that $(\hat{\alpha}_o \hat{\varepsilon})^2 = \hat{\varepsilon}^2$, $(\hat{\alpha} \hat{p})^2 = \hat{p}^2$, where $\hat{\alpha}_o = \begin{pmatrix} \hat{\sigma}_o & 0 \\ 0 & \hat{\sigma}_o \end{pmatrix}$; $\hat{\alpha} = \begin{pmatrix} 0 & \hat{\sigma} \\ \hat{\sigma} & 0 \end{pmatrix}$; $\hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_o & 0 \\ 0 & -\hat{\sigma}_o \end{pmatrix}$ are Dirac's matrices and $\hat{\sigma}_o, \hat{\sigma}$ are Pauli matrices, the equation (2.4) can also be represented in the matrix form of the Klein-Gordon-like equation without mass:

$$\left[(\hat{\alpha}_o \hat{\varepsilon})^2 - c^2 (\hat{\alpha} \hat{p})^2 \right] \Phi = 0, \quad (2.5)$$

Taking also into account that in case of photon we have $\omega = \varepsilon/\hbar$ and $k = p/\hbar$, from (2.5), using (2.1), we obtain $\varepsilon = cp$, as it is has place for a photon. Therefore we can consider the Φ - wave function of the equation (2.5) both as EM wave and as a photon.

Factorizing (2.5) and multiplying it from left on the Hermitian-conjugate function Φ^+ we get:

$$\Phi^+ (\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) (\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) \Phi = 0, \quad (2.6)$$

The equation (2.6) may be disintegrated on two Dirac-like equations without mass:

$$\Phi^+ (\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) = 0, \quad (2.7')$$

$$(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) \Phi = 0, \quad (2.7'')$$

It is not difficult to show that only in the case when we choose the Φ -matrix in the following form:

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \Phi^+ = (E_x \ E_z \ -iH_x \ -iH_z), \quad (2.8)$$

the equations (2.7) are the right Maxwell equations of the electromagnetic waves: retarded and advanced. Actually using (2.8) and putting it in (2.7) we obtain:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \end{array} \right. , \quad (2.9') \quad \left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \end{array} \right. , \quad (2.9'')$$

For waves of any other direction the same results can be obtained by the cyclic transposition of the indexes and by the canonical transformation of matrices and wave functions (see chapter 3).

We will further conditionally name each of (2.7) equations the linear semi-photon equations, remembering that it was obtained by division of one wave equation of a photon into two equations of the electromagnetic waves: retarded and advanced.

3.0. Twirl transformation of electromagnetic wave

The transformation of the linear wave to the curvilinear (briefly – “twirl transformation”) can be conditionally represented as following expression:

$$\hat{R}\Phi \rightarrow \Psi, \quad (3.1)$$

where \hat{R} is the operator of trajectory transformation of EM wave from linear to curvilinear, the Φ is the wave function, defined by matrix (2.8), which satisfies the equations (2.5) and (2.7), and Ψ is some wave function:

$$\Psi = \begin{pmatrix} E'_x \\ E'_z \\ iH'_x \\ iH'_z \end{pmatrix}, \quad (3.2)$$

which appears after non-linear transformation; here $\Psi(y) = \{E'_x, E'_z, H'_x, H'_z\}$ are electromagnetic field vectors after twirl transformation.

As it is known, the description of vector transition from linear to curvilinear trajectory is fully described by differential geometry (Eisenhart, 1960). Note also that mathematically this transition is equivalent to the vector transition from flat space to the curvilinear space, which is described by Riemann geometry.

In connection to this let us remind that the Pauli matrices as well as the photon matrices are the space rotation operators – 2-D and 3-D correspondingly (Ryder, 1987).

3.1. The twirl transformation description in differential geometry

Let the plane-polarized wave, which has the field vectors (E_x, H_z) , be twirled with some radius r_p in the plane (X', O', Y') of a fixed co-ordinate system (X', Y', Z', O') so that E_x is parallel to the plane (X', O', Y') and H_z is perpendicular to it (figs 1):

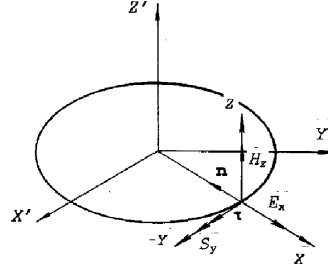


Fig. 1

According to Maxwell (Jackson, 1999) the displacement current in the equation (2.9) is defined by the expression:

$$\vec{j}_{dis} = \frac{1}{4\pi} \frac{\partial \vec{E}}{\partial t}, \quad (3.3)$$

The above electrical field vector \vec{E} , which moves along the curvilinear trajectory (let it have direction from the center), can be written in the form:

$$\vec{E} = -E \cdot \vec{n}, \quad (3.4)$$

where $E = |\vec{E}|$, and \vec{n} is the normal unit-vector of the curve (having direction to the center). The derivative of \vec{E} can be represented as:

$$\frac{\partial \vec{E}}{\partial t} = -\frac{\partial E}{\partial t} \vec{n} - E \frac{\partial \vec{n}}{\partial t}, \quad (3.5)$$

Here the first term has the same direction as \vec{E} . The existence of the second term shows that at the twirling of the wave the additional displacement current appears. It is not difficult to show that it has direction, tangential to the ring:

$$\frac{\partial \vec{n}}{\partial t} = -v_p \kappa \vec{\tau}, \quad (3.6)$$

where $\vec{\tau}$ is the tangential unit-vector, $v_p \equiv c$ is the electromagnetic wave velocity, $\kappa = \frac{1}{r_p}$ is the curvature of the trajectory and r_p is the curvature radius. Thus, the displacement current of the plane wave, moving along the ring, can be written in the form:

$$\vec{j}_{dis} = -\frac{1}{4\pi} \frac{\partial E}{\partial t} \vec{n} + \frac{1}{4\pi} \omega_p E \cdot \vec{\tau}, \quad (3.7)$$

where $\omega_p = \frac{m_p c^2}{\hbar} = \frac{v_p}{r_p} \equiv c\kappa$ we will name the curvature angular velocity, $\epsilon_p = m_p c^2$ is

photon energy, m_p is some mass, corresponding to the energy ϵ_p , $\vec{j}_n = \frac{1}{4\pi} \frac{\partial E}{\partial t} \vec{n}$ and

$\vec{j}_\tau = \frac{\omega_p}{4\pi} E \cdot \vec{\tau}$ are the normal and tangent components of the current of the twirled electromagnetic wave, correspondingly. Thus:

$$\vec{j}_{dis} = \vec{j}_n + \vec{j}_\tau, \quad (3.8)$$

The currents \vec{j}_n and \vec{j}_τ are always mutually perpendicular, so that we can write them in the complex form:

$$j_{dis} = j_n + ij_\tau, \quad (3.8')$$

where $j_\tau = \frac{\omega_p}{4\pi} E$. Thus the tangent current appearance causes the appearance of imaginary unit.

From the above we can also assume that *the appearance of imaginary unit in the quantum mechanics is tied with the tangent current appearance.*

3.2. The twirl transformation description in Riemann geometry

We can consider conditionally the Maxwell wave equations (2.7) with wave function (2.8) as Dirac equation without mass.

The generalization of the Dirac equation on the curvilinear (Riemann) geometry is connected with the parallel transport of the spinor in the curvilinear space (Fock, 1929a,b; Fock and Ivanenko, 1929; Van der Waerden, 1929; Schroedinger, 1932; Infeld und Van der Waerden, 1933; Goenner, 2004). For the generalization of the Dirac (without mass) equation on the Riemann geometry it is enough to replace the usual derivative $\partial_\mu \equiv \partial / \partial x_\mu$ (where x_μ are the co-ordinates in the 4-space) with the covariant derivative:

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad (3.9)$$

where $\mu = 0, 1, 2, 3$ are the summing indices, and Γ_μ is the analogue of Christoffel's symbols in the case of the spinor theory, called Ricci symbols (or connection coefficients).

In the theory it shown that $\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_i p_i + i \hat{\alpha}_0 p_0$, where p_i and p_0 are the real values. It is not difficult to see that the tangent current j_τ corresponds to the Ricci connection coefficients (symbols) Γ_μ .

When a spinor moves along the straight line, all the symbols $\Gamma_\mu = 0$, and we have a usual derivative. But if a spinor moves along the curvilinear trajectory, not all the Γ_μ are equal to zero and a supplementary term appears.

Typically, the last one is not the derivative, but it is equal to the product of the spinor itself with some coefficient Γ_μ , which is increment in spinor. Since, according to the general theory (Sokolov and Ivanenko, 1952), the increment in spinor Γ_μ has the form and the dimension of the energy-momentum 4-vector, it is logical to identify Γ_μ with 4-vector of energy-momentum of the photon electromagnetic field:

$$\Gamma_\mu = \{\mathcal{E}_p, c\vec{p}_p\}, \quad (3.10)$$

where \mathcal{E}_p and p_p is the photon energy and momentum (not the operators). In other words we have:

$$\hat{\alpha}_\mu \Gamma_\mu = \hat{\alpha}_0 \mathcal{E}_p + \vec{\hat{\alpha}} \vec{p}_p, \quad (3.11)$$

Taking into account that according to energy conservation law $\hat{\alpha}_0 \mathcal{E}_p + \vec{\hat{\alpha}} \vec{p}_p = \pm \hat{\beta} m_p c^2$, it is not difficult to see that the supplementary term contains a twirled wave mass.

4.0. The equations of twirled electromagnetic wave

4.1. Klein-Gordon-like equation of twirling photon

As it follows from previous sections due to the curvilinear motion of the electromagnetic wave, some additional terms $K = \hat{\beta} m_p c^2$, corresponding to the tangent components of the displacement current, will appear in the equation (2.6), so that from (2.6) we have:

$$\left(\hat{\alpha}_0 \hat{\mathcal{E}} - c \hat{\alpha} \cdot \hat{p} - K \right) \left(\hat{\alpha}_0 \hat{\mathcal{E}} + c \hat{\alpha} \cdot \hat{p} + K \right) \Psi = 0, \quad (4.1)$$

Thus, in the case of the curvilinear motion of the electromagnetic fields of photon, instead of the equation (2.6) we obtain the Klein-Gordon-like equation with mass (Schiff, 1955):

$$\left(\hat{\mathcal{E}}^2 - c^2 \hat{p}^2 - m_p^2 c^4 \right) \Psi = 0, \quad (4.2)$$

As we see the Ψ -function, which appears after electromagnetic wave twirling and satisfies the equation (4.2), is not identical to the Φ -function before twirling. The Φ -function is the classical

linear electromagnetic wave field, which satisfies the wave equation (2.7); in the same time the Ψ - function is the non-classical curvilinear electromagnetic wave field, which satisfies the Klein-Gordon-like equation (4.2).

As it is known in quantum physics the Klein-Gordon equation is considered as the scalar field equation. But obviously *the Klein-Gordon-like equation (4.2), whose wave function is 4×1 - matrix with electromagnetic field components, cannot have the sense of the scalar field equation.* Actually, let us analyze the objects, which this equation describes.

From the Maxwell equations follows that each of the components $E_x, E_y, E_z, H_x, H_y, H_z$ of vectors of an electromagnetic field \vec{E}, \vec{H} submits to the same form of the scalar wave equations. In the case of the linear wave all field components are independent. Here by study of one of the \vec{E}, \vec{H} vector's components, we can consider the vector field as scalar. But after the twirl transformation, i.e. in the framework CWED, when a tangential current appears, we cannot proceed to the scalar theory, since the components of a vector \vec{E} , as it follows from the condition (Maxwell law) $\vec{\nabla} \cdot \vec{E} = \frac{4\pi}{c} \vec{c}^0 \cdot \vec{j}$

(where \vec{c}^0 is the unit vector of wave velocity) are not independent functions.

Therefore, although the Klein - Gordon equation for scalar wave function describes a massive particle with spin zero (spinless boson), the equations (4.2) concerning electromagnetic wave functions (3.2), which appears after curvilinear transformation, represents the equation of the vector particle with rest mass m_p and with spin one. In this sense this equation play the role of the Proca equation. To except this difficulty with the name we will name it as the **twirled photon equation**.

4.2. The equation of the twirled semi-photon

Using the disintegration (4.1) we can obtain from **twirled photon equation** (4.2) the equations of the twirled electromagnetic wave – advanced and retarded:

$$\left[(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} m_p c^2 \right] \psi = 0, \quad (4.3')$$

$$\psi^+ \left[(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} m_p c^2 \right] = 0, \quad (4.3'')$$

where $\psi = \{E_x, E_z, H_x, H_z\}$ is some EM wave function, which we will be name further the **twirled semi-photon equations**. And the above transition from (4.2) to (4.3) we can conditionally name a “**symmetry breaking transformation**”.

Now we will analyse the particularities of the equations (4.3). It is not difficult to see that the lasts are similar to Dirac electron equations. But note that instead of electron mass m_e , equations (4.3) contain the twirled photon mass m_p . The question arises what type of EM particles the equations (4.3) describe?

In the case of electron-positron pair production it must be $m_p = 2m_e$ so that from (4.3) we have:

$$\left[(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + 2\hat{\beta} m_e c^2 \right] \psi = 0, \quad (4.4')$$

$$\psi^+ \left[(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - 2\hat{\beta} m_e c^2 \right] = 0, \quad (4.4'')$$

Obviously after the twirled photon breaking, i.e. after the chargeless twirled photon is divided into two charged semi-photon, the plus and minus charged particles acquire the electric fields, and each particle begins to move in the field of another. In order to become the independent (i.e. free) particles, they must be drawn away one from the other on great distance

Therefore, the equations, which arise after the twirled photon division, cannot be the free positive and negative (electron and positron) particle equations, but the particle equations with the external field.

In this case during the charged particles move away one from another the energy, which correspond to the energy of the electric field creation, must be expended. In fact, being the particles combined, the system doesn't have any field (fig. 3). At very small distance they create the dipole field. And at a distance, much more than the particle radius, the plus and minus particles acquire the full electric fields. As it is known (Jackson, 1999), the potential V_p of both plus and minus charges in the point P is defined as:

$$V_p = \frac{e}{4\pi} \left(\frac{1}{r} - \frac{1}{r + d \cos \theta} \right), \quad (4.5)$$

where $\pm e$ are the dipole charges, d is the distance between the charges, and θ is the angle between axes and radius-vector of plus particle. For $d = 0$ we have $V_p = 0$. For $d \rightarrow \infty$ we obtain, as the limit case, the Coulomb potential for each free particles:

$$\lim_{d \rightarrow \infty} V_p = \frac{1}{4\pi} \frac{e}{r}, \quad (4.6)$$

Thus during the breaking process the particle charges appear. For the particle, removed to infinity, the work against the attractive forces needed to be fulfilled:

$$\varepsilon_{rel} = \oint e V_p d\nu = \frac{1}{4\pi} \oint \frac{e^2}{r} d\nu, \quad (4.7)$$

Obviously, the external particles field defines this work, so that the release energy is the field production energy and in the same time it is the annihilation energy. Therefore, due to energy conservation law this energy value for each particle must be equal $\varepsilon_{rel} = m_e c^2$.

So, the equations (4.3) we can write in the following form:

$$\left[(\hat{\alpha}_0 \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} m_e c^2 + \hat{\beta} m_e c^2 \right] \psi = 0, \quad (4.8')$$

$$\psi^+ \left[(\hat{\alpha}_0 \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} m_e c^2 - \hat{\beta} m_e c^2 \right] = 0, \quad (4.8'')$$

Using the linear equation for description of the energy conservation law, we can write:

$$\pm \hat{\beta} m_e c^2 = -\varepsilon_{ex} - c \hat{\alpha} \vec{p}_{ex} = -e \varphi_{ex} - e \hat{\alpha} \vec{A}_{ex}, \quad (4.9)$$

where “ex” means “external”. Putting (4.9) in (4.8) we obtain the Dirac equation with external field:

$$\left[\hat{\alpha}_0 (\hat{\varepsilon} \mp \varepsilon_{ex}) + c \hat{\alpha} \cdot (\hat{p} \mp \vec{p}_{ex}) + \hat{\beta} m_e c^2 \right] \psi = 0, \quad (4.10)$$

which at $d \rightarrow \infty$ give the Dirac free plus and minus particle equations:

$$\left[(\hat{\alpha}_0 \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} m_e c^2 \right] \psi = 0, \quad (4.11')$$

$$\psi^+ \left[(\hat{\alpha}_0 \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} m_e c^2 \right] = 0, \quad (4.11'')$$

From above some interesting consequences follow:

1. before breaking the twirled photon is not an absolutely neutral particle, but a dipole; therefore, it must have the dipole moment.

2. the formula (4.9) shows that in CWED the mass is not equivalent to the energy, but to the 4-vector of the energy-momentum; from this follows that in CWED the energy has the kinetic origin.

3. in framework of CWED for free term of particle equation the following expression take place:

$$\pm \hat{\beta} m_e c^2 = -\varepsilon_{in} - c \hat{\alpha} \vec{p}_{in} = -e \varphi_{in} - e \hat{\alpha} \vec{A}_{in}, \quad (4.12)$$

where “in” means “internal”. In other words the values $(\varepsilon_{in}, \vec{p}_{in})$ describe the inner field, and the values $(\varepsilon_{ex}, \vec{p}_{ex})$ the external field of electron-positron particles. When we consider the electron particle from great distance, the field $(\varepsilon_{in}, \vec{p}_{in})$ works as the mass, and the term $(\varepsilon_{ex}, \vec{p}_{ex})$ describes the external electromagnetic field (and we have linear Dirac equations of particles). Inside the electron the term $(\varepsilon_{in}, \vec{p}_{in})$ is needed for the detailed description of the inner field of particle, which characterizes the particle parts interaction (as it is shown below, this term carries to non-linear equation of particle).

Using (3.2) we obtain electromagnetic form of the equations (4.11):

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = ij_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -ij_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = ij_x^m \end{array} \right. , (4.13') \quad \left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = ij_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -ij_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = ij_x^m \end{array} \right. , (4.13'')$$

where

$$\begin{aligned} j^e &= i \frac{\omega_e}{4\pi} E = i \frac{c}{4\pi r_e} E \\ j^m &= i \frac{\omega_e}{4\pi} H = i \frac{c}{4\pi r_e} H \end{aligned} , \quad (4.14)$$

are the “imaginary” currents, in which $\omega = \frac{2m_e c^2}{\hbar}$, and $r_e = \frac{\hbar}{2m_e c}$ is the radius of twirling of EM wave (and it is also the half of Compton wavelength of the electron). As we see the equations (4.13') and (4.13'') are Maxwell equations with imaginary electric and magnetic currents. As it is known the existence of the magnetic current \vec{j}^m doesn't contradict to the quantum theory (see the Dirac theory of the magnetic monopole (Dirac, 1931)). In our case of the plane polarized wave (see figs. 2 and 3) the magnetic currents are equal to zero.

Thus, the equations (4.11) are Maxwell equations with imaginary tangential currents and simultaneously they are the Dirac equation of electron.

5.0. Analysis of the free electron equation solution from EM point of view

In accordance with the above results the electromagnetic form of the solution of the Dirac free electron equation must be a twirled electromagnetic wave.

If this supposition is actually correct, for the y -direction photon two solutions must exist:

1) for the wave, twirled around the OZ -axis

$${}^{oz}\psi = \begin{pmatrix} E_x \\ 0 \\ 0 \\ iH_z \end{pmatrix} = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \\ \psi_4 \end{pmatrix}, \quad (5.1)$$

and 2) for the wave, twirled around the OX -axis

$${}^{ox}\psi = \begin{pmatrix} 0 \\ E_z \\ iH_x \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \psi_2 \\ \psi_3 \\ 0 \end{pmatrix}, \quad (5.2)$$

The ψ -functions (5.1) and (5.2) as the solutions of the equations (4.11) must have the same expressions as the Dirac electron theory solutions (Schiff, 1955). Let us analyze the Dirac electron theory solutions from CWED point of view.

It is known (Schiff, 1955) that the solution of the Dirac free electron equation (2.1) has the form of the plane wave:

$$\psi_j = B_j \exp\left(-\frac{i}{\hbar}(\epsilon t - \vec{p}\vec{r})\right), \quad (5.3)$$

where $j = 1, 2, 3, 4$; $B_j = b_j e^{i\phi}$; the amplitudes b_j are the numbers and ϕ is the initial wave phase. The functions (5.3) are the eigenfunctions of the energy-momentum operators, where \mathcal{E} and \vec{p} are the energy-momentum eigenvalues. Here for each \vec{p} , the energy \mathcal{E} has either positive or negative values according to the energy-momentum conservation law equation $\mathcal{E}_{\pm} = \pm \sqrt{c^2 \vec{p}^2 + m_e^2 c^4}$.

For \mathcal{E}_+ we have two linear-independent set of four orthogonal normalizing amplitudes:

$$1) B_1 = -\frac{cp_z}{\mathcal{E}_+ + m_e c^2}, B_2 = -\frac{c(p_x + ip_y)}{\mathcal{E}_+ + m_e c^2}, B_3 = 1, B_4 = 0, \quad (5.4)$$

$$2) B_1 = -\frac{c(p_x - ip_y)}{\mathcal{E}_+ + m_e c^2}, B_2 = \frac{cp_z}{\mathcal{E}_+ + m_e c^2}, B_3 = 0, B_4 = 1, \quad (5.5)$$

and accordingly for \mathcal{E}_- :

$$3) B_1 = 1, B_2 = 0, B_3 = \frac{cp_z}{-\mathcal{E}_- + m_e c^2}, B_4 = \frac{c(p_x + ip_y)}{-\mathcal{E}_- + m_e c^2}, \quad (5.6)$$

$$4) B_1 = 0, B_2 = 1, B_3 = \frac{c(p_x - ip_y)}{-\mathcal{E}_- + m_e c^2}, B_4 = -\frac{cp_z}{-\mathcal{E}_- + m_e c^2}, \quad (5.7)$$

Each of these four solutions (Schiff, 1955) can be normalized by its multiplication by normalization factor:

$$\kappa = \left[1 + \frac{c^2 \vec{p}^2}{(\mathcal{E}_+ + m_e c^2)^2} \right]^{-\frac{1}{2}},$$

which gives $\Psi^+ \Psi = 1$

Let's discuss these results.

1) The existing of two linear independent solutions corresponds with two independent orientations of the electromagnetic wave vectors and gives the unique logic explanation for this fact.

2) Since $\Psi = \Psi(y)$, we have $p_x = p_z = 0$, $p_y = m_e c$ and for the field vectors we obtain:

from (4.4) and (4.5) for "positive" energy

$$B_+^{(1)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi}, B_+^{(2)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi}, \quad (5.8)$$

and from (4.6) and (4.7) for "negative" energy:

$$B_-^{(1)} = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ b_4 \end{pmatrix} \cdot e^{i\phi}, B_-^{(2)} = \begin{pmatrix} 0 \\ b_2 \\ b_3 \\ 0 \end{pmatrix} \cdot e^{i\phi}, \quad (5.9)$$

which exactly correspond to (5.1) and (5.2).

3) Calculate the correlations between the components of the field vectors. Putting $\phi = \frac{\pi}{2}$ for $\mathcal{E}_+ = m_e c^2$ and $\mathcal{E}_- = -m_e c^2$ we obtain correspondingly:

$$B_+^{(1)} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ i \cdot 1 \\ 0 \end{pmatrix}, B_+^{(2)} = \begin{pmatrix} -\frac{1}{2} \\ 0 \\ 0 \\ i \cdot 1 \end{pmatrix}, \quad (5.10)$$

$$B_-^{(1)} = \begin{pmatrix} i \cdot 1 \\ 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix}, \quad B_-^{(2)} = \begin{pmatrix} 0 \\ i \cdot 1 \\ \frac{1}{2} \\ 0 \end{pmatrix}, \quad (5.11)$$

Obviously the imaginary unit in these solutions indicates that the field vectors \vec{E} and \vec{H} are mutually orthogonal.

Also we see that the electric field amplitude is two times less, than the magnetic field amplitude. This fact shows that the electromagnetic field's values, which correspond to the Dirac equation solution, are different contrary to fields of the linear wave of the Maxwell theory, where $\vec{E} = \vec{H}$. (It can be shown that this result provides the electron stability).

4) It is easy to show that the electromagnetic form of the solution of the Dirac equation is the standing wave. Really in case of the circle-twirled wave we have $\vec{p} \perp \vec{r}$ and therefore $\vec{p} \cdot \vec{r} = 0$; then instead (4.3) we obtain:

$$\psi_j = b_j \exp\left(-\frac{i}{\hbar} \varepsilon t\right), \quad (5.12)$$

5) According with the Euler formula $e^{i\varphi} = \cos\varphi + i\sin\varphi$ the solution of the Dirac equation (5.12) describes a circle, as it corresponds to our theory.

6) Let's calculate the normalization factor, substituting: $p = m_e c, \varepsilon = m_e c^2$:

$$\kappa = \left(\frac{5}{4}\right)^{-\frac{1}{2}}, \quad (5.13)$$

and compare it with normalization factor, which is received from the electromagnetic representation of the theory. In view of that the electric field is twice less of magnetic field, the energy density of twirled semi-photon will be equal:

$$W_{s-ph} = \frac{1}{8\pi} (E_{s-ph}^2 + H_{s-ph}^2) = \frac{1}{8\pi} \left[\left(\frac{1}{2} H_{s-ph}^2 \right) + H_{s-ph}^2 \right] = \frac{1}{8\pi} \frac{5}{4} H_{s-ph}^2, \quad (5.14)$$

Using non-normalized expression for the wave function:

$$\psi_j = B_0 B_j e^{i(\vec{k}\vec{r} - \omega t)} = B_0 \begin{pmatrix} 0 \\ i \frac{1}{2} \\ 1 \\ 0 \end{pmatrix} e^{i(\vec{k}\vec{r} - \omega t)}, \quad (5.15)$$

(where B_0 is some constant, generally dimensional), and using also the Hermitian-conjugate function:

$$\psi_j^+ = B_0 B_j^+ e^{-i(\vec{k}\vec{r} - \omega t)} = B_0 \begin{pmatrix} 0 & -i \frac{1}{2} & 1 & 0 \end{pmatrix} e^{-i(\vec{k}\vec{r} - \omega t)}, \quad (5.16)$$

for field energy we will receive the following expression:

$$W = \frac{1}{8\pi} \psi_j^+ \psi_j = \frac{1}{8\pi} \cdot \frac{5}{4} \cdot B_0^2, \quad (5.17)$$

which precisely corresponds to the quantum theory result.

6.0. Particularities of wave function of electron equation

As is known the fields of a photon are vectors, transforming as elements of group (O3). The spinor fields of the Dirac equation are transformed as elements of group (SU2). As it is shown by L.H. Ryder (Ryder, 1987) and others, two spinor transformations correspond to one transformation of a vector. For this reason the spinors are also named "semi-vectors" or "tensors of half rank" (Goenner, 2004; Sokolov & Ivanenko, 1952).

From above following that the twirling and breaking of the twirled photon waves corresponds to transition from usual linear Maxwell equation to the EM curvilinear wave equation with an imaginary tangential currents (i.e. to the EM Dirac equation). Obviously, the transformation properties of electromagnetic fields at this transition change. Just as the wave functions of the Dirac equation (i.e. spinors) submit to transformations of group (SU2), the semi-photon fields must submit to the same transformations.

Let us try now to specify the differences between electromagnetic fields $\{E'_x, E'_z, H'_x, H'_z\}$ of the Ψ -function of twirled photon and electromagnetic fields $\{E_x, E_z, H_x, H_z\}$ of ψ -function of twirled semi-photon. Taking into account that we have the same mathematical equations both for the CWED Dirac equation and the Dirac electron equation, we can affirm that these transformation features coincide with the same features of the spinor (Ryder, 1987; Gottfried & Weisskopf, 1984).

The spinor invariant transformation has the form:

$$\psi' = U\psi, \quad (6.1)$$

where the operator of transformation is entered as follows:

$$U(\vec{n}\theta) = \cos\frac{1}{2}\theta - i\vec{n} \cdot \vec{\sigma}' \sin\frac{1}{2}\theta, \quad (6.2)$$

where \vec{n} is the unit vector of an axis, θ is a rotation angle around this axis and $\vec{\sigma}' = (\sigma'_x, \sigma'_y, \sigma'_z)$ is the spin vector.

The rotation matrix (6.2) possesses a remarkable property. If the rotation occurs on the angle $\theta = 2\pi$ around any axis (therefore occurs the returning to the initial system of reference) we find, that $U = -1$, instead of $U = 1$ as it was possible to expect. In other words, the state vector of system with spin half in usual three-dimensional space has two-valuedness and passes to itself only after turn to the angle 4π .

This result can be explained only if we suppose that the *EM electron is the twirled half-period of a twirled photon particle, and therefore needs to be rotated twice to return to the initial state*. In other words, the twirled semi-photon is the twirled half-period of the photon.

Taking into account the above results the solution of the EM electron equation (i.e. Dirac equation in the EM form) we can name "electromagnetic spinor". In other words the electromagnetic spinor is the semi-period of twirling EM wave. Thus, the transformation of the "linear" electromagnetic wave into curvilinear wave and its symmetry breaking produces the electromagnetic spinors.

7.0. Electromagnetic Non-linear Electron Equation and its Lagrangian

7.1. The EM nonlinear electron equation

Obviously the curvilinearity of photon or semi-photon motion must be described by non-linear equation. From this it follows that the CWED equation of EM electron must be the non-linear field equation. Let us find it electromagnetic and quantum forms.

The stability of twirled semi-photon is possible only by the semi-photon part's self-action. Using (4.12) from (4.11) we will obtain the following non-linear equation:

$$\left[\hat{\alpha}_0 (\hat{\varepsilon} - \varepsilon_{in}) + c\hat{\alpha} \cdot (\hat{p} - \vec{p}_{in}) \right] \psi = 0, \quad (7.1)$$

where the inner energy and momentum can be expressed, using the inner energy density U and momentum density \vec{g} (or Poynting vector \vec{S}) of EM wave:

$$\varepsilon_{in} = \int_0^\tau U d\tau = \frac{1}{8\pi} \int_0^\tau (\vec{E}^2 + \vec{H}^2) d\tau, \quad (7.2)$$

$$\vec{p}_{in} = \int_0^\tau \vec{g} d\tau = \frac{1}{c^2} \int_0^\tau \vec{S} d\tau = \frac{1}{4\pi} \int_0^\tau [\vec{E} \times \vec{H}] d\tau, \quad (7.3)$$

putting the upper limit τ to be variable.

Substituting of the expression (7.2) and (7.3) to the EM electron equation, we obtain *the non-linear integral-differential equation, which is, as we suppose, the searched form of the non-linear equation*, which describes the EM-electron in both electromagnetic and concurrent quantum forms.

To show, that the equation (7.1) can actually pretend to the role of the equation of non-linear electrodynamics of the electron EM particle, we find its approximate quantum form.

Using EM form of ψ - function, it is easy to prove that the quantum forms of U and \vec{S} are:

$$U = \frac{1}{8\pi} \psi^+ \hat{\alpha}_0 \psi, \quad (7.4)$$

$$\vec{S} = -\frac{c}{8\pi} \psi^+ \hat{\alpha} \psi = c^2 \vec{g}, \quad (7.5)$$

Taking into account that the free electron Dirac equation solution is the plane wave:

$$\psi = \psi_0 \exp[i(\omega t - ky)], \quad (7.6)$$

we can write (7.2) and (7.3) in the next approximate form:

$$\varepsilon_p = U \Delta\tau = \frac{\Delta\tau}{8\pi} \psi^+ \hat{\alpha}_0 \psi, \quad (7.7)$$

$$\vec{p}_p = \vec{g} \Delta\tau = \frac{1}{c^2} \vec{S} \Delta\tau = -\frac{\Delta\tau}{8\pi c} \psi^+ \hat{\alpha} \psi, \quad (7.8)$$

where $\Delta\tau$ is the volume, which contain the main part of the twirled semi-photon energy. Then the approximate form of the equation (7.3) will be following:

$$\frac{\partial \psi}{\partial t} - c \hat{\alpha} \vec{\nabla} \psi + i \frac{\Delta\tau}{8\pi c} (\psi^+ \hat{\alpha}_0 \psi - \hat{\alpha} \psi^+ \hat{\alpha} \psi) \psi = 0, \quad (7.9)$$

It is not difficult to see that the equation (7.9) is the non-linear equation of the same type as non-linear Heisenberg equation (Heisenberg, 1966; Paper translation collection, 1959):

$$\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \frac{1}{2} l [\gamma_\mu \psi (\bar{\psi} \gamma_\mu \psi) + \gamma_\mu \gamma_3 \psi (\bar{\psi} \gamma_\mu \gamma_3 \psi)] = 0, \quad (7.10)$$

if instead of α -set Dirac matrices we will use γ -set matrices (here l is some positive constant). The non-linear equation (7.10) was postulated and investigated by Heisenberg et. al. as the unitary quantum field theory equation. Contrary to the last one, the equation (7.9) is obtained by logical and correct way and the self-action constant l appeared in (7.9) automatically. As it is known in the framework of this non-linear unitary field theory some substantial achievements were made.

7.2. The Lagrangian of the nonlinear electron theory

The Lagrangian of the Dirac electron theory of linear type in quantum form is (Schiff, 1955):

$$L_D = \psi^+ (\hat{\varepsilon} + c \hat{\alpha} \hat{p} + \hat{\beta} m_e c^2) \psi, \quad (7.11)$$

It is not difficult to find its electromagnetic form:

$$L_D = \frac{\partial U}{\partial t} + \text{div } \vec{S} - i \frac{\omega}{8\pi} (\vec{E}^2 - \vec{H}^2), \quad (7.12)$$

(Note that in the case of the variation procedure we must distinguish the complex conjugate field vectors \vec{E}^* , \vec{H}^* and \vec{E} , \vec{H}).

The Lagrangian of non-linear theory is not difficult to obtain from the Lagrangian (7.11) using the method by which we found the nonlinear equation. By substituting (5.1) we obtain:

$$L_N = \psi^+ (\hat{\varepsilon} - c \hat{\alpha} \cdot \hat{p}) \psi + \psi^+ (\varepsilon_{in} - c \hat{\alpha} \cdot \vec{p}_{in}) \psi, \quad (7.13)$$

We suppose that the expression (7.13) represents the common form of the Lagrangian of the non-linear electron theory. In order to compare (7.13) with the known results of classical and quantum physics let us find the approximate electromagnetic and quantum forms of this Lagrangian.

Using (7.7) and (7.8) we can represent (7.11) in the following quantum form:

$$L_N = i\hbar \left[\frac{\partial}{\partial t} \left[\frac{1}{2} (\psi^+ \psi) \right] - c \text{div} (\psi^+ \hat{\alpha} \psi) \right] + \frac{\Delta\tau}{8\pi} \left[(\psi^+ \psi)^2 - (\psi^+ \hat{\alpha} \psi)^2 \right], \quad (7.14)$$

To obtain the EM form of (7.14) we initially pass on to normalized Ψ -function, using the expression $L'_N = \frac{1}{8\pi m_e c^2} L_N$. Then we transform (7.13), using equations (7.4) and (7.5), and obtain from (7.14) the following approximate electromagnetic form:

$$L'_N = i \frac{\hbar}{2m_e} \left(\frac{1}{c^2} \frac{\partial U}{\partial t} + \text{div } \vec{g} \right) + \frac{\Delta \tau}{m_e c^2} (U^2 - c^2 \vec{g}^2), \quad (7.15)$$

It is not difficult to transform the second term, using the known identity of electrodynamics:

$$(8\pi)^2 (U^2 - c^2 \vec{g}^2) = (\vec{E}^2 + \vec{H}^2)^2 - 4(\vec{E} \times \vec{H})^2 = (\vec{E}^2 - \vec{H}^2)^2 + 4(\vec{E} \cdot \vec{H})^2, \quad (7.16)$$

Thus, taking into account that $L_D = 0$ and using (7.12) and (7.16), we obtain from (7.15) the following expression:

$$L'_N = \frac{1}{8\pi} (\vec{E}^2 - \vec{H}^2) + \frac{\Delta \tau}{(8\pi)^2 m_e c^2} \left[(\vec{E}^2 - \vec{H}^2)^2 + 4(\vec{E} \cdot \vec{H})^2 \right], \quad (7.17)$$

As we see, the approximate form of the Lagrangian of the nonlinear equation of the twirled electromagnetic wave contains only the invariants of the Maxwell theory and is similar to the known Lagrangian of the photon-photon interaction (Akhiezer and Berestetskii, 1965).

Let us now analyze the quantum form of the Lagrangian density (7.17). The equation (7.12) can be written in the form:

$$L_Q = \Psi^+ \hat{\alpha}_\mu \partial_\mu \Psi + \frac{\Delta \tau}{8\pi} \left[(\Psi^+ \hat{\alpha}_0 \Psi)^2 - (\Psi^+ \hat{\alpha} \Psi)^2 \right], \quad (7.18)$$

It is not difficult to see that the electrodynamics correlation (7.16) in quantum form has the form of the known Fierz identity (Cheng and Li, 1984; 2000):

$$(\Psi^+ \hat{\alpha}_0 \Psi)^2 - (\Psi^+ \hat{\alpha} \Psi)^2 = (\Psi^+ \hat{\alpha}_4 \Psi)^2 + (\Psi^+ \hat{\alpha}_5 \Psi)^2, \quad (7.19)$$

Using (7.19) from (7.18) we obtain:

$$L_Q = \Psi^+ \hat{\alpha}_\mu \partial_\mu \Psi + \frac{\Delta \tau}{8\pi} \left[(\Psi^+ \hat{\alpha}_4 \Psi)^2 - (\Psi^+ \hat{\alpha}_5 \Psi)^2 \right], \quad (7.20)$$

The Lagrangian (7.20) coincides with the Nambu and Jona-Lasinio Lagrangian (Nambu and Jona-Lasinio, 1961; 1961a), which is the Lagrangian density of the relativistic superconductivity theory. As it is known this Lagrangian density is used for the solution of the problem of the elementary particles mass appearance by the mechanism of the vacuum symmetry spontaneous breakdown (it corresponds also to the Cooper's pair production process in the superconductivity theory).

Note again that in our theory, the breakdown of symmetry also takes place when a mass of particles appears within twirling and breaking of photon.

8.0. About peculiarities of CWED as the non-linear theory

It is not difficult to see that CWED disclose two types of non-linearity. The first type is connected with postulate 6 of CWED about the motion of EM wave along curvilinear closed trajectories. The curvilinearity, as a deviation from linearity, is possible to consider as one of kinds of non-linearity. But in our case these non-linear trajectories concern to concrete kind: they are created and described by harmonic functions and by their superpositions. It allows to describe this non-linearity by the linear equations.

Really, the motion along a circle can be presented as the sum of two linear harmonic oscillations. The sum of greater number of oscillations leads to the multiform (including, spatial) curvilinear trajectories, known as Lissajous figures. Apparently, in this connection all these non-linearities are conveniently and simply described by complex functions (more detail see chapter 8). It is possible to assume that the Fourier apparatus of the analysis and synthesis of functions reflects such opportunity of the linear description of curves, which can be described by the sum of the linear harmonic oscillations.

In this case it is possible in the existence of Fourier theory to see the reflection of the reality, described by CWED. Since the Fourier theory can be used only in the case of linear functions, obviously, this "harmonic curvilinearity" allows in these conditions to consider the CWED to be the

linear theory, i.e. the theory, in which as well as in the quantum field theory, the principle of superposition is strictly carried out.

But, on the other hand, as we saw, the twirling of EM waves results also in other type of non-linearity. Really, we deal here not only with trajectories, but with the fields, which "are attached" to this trajectory by strictly defined manner. During formation of EM particles, i.e. as a result of bending of trajectory of an EM wave, inside of its volume the field configuration varies. This enters into the equations the non-linear terms, which are presented neither in classical electrodynamics, nor in the linear quantum field theory. The splitting up of the twirled photon into two twirled half-period even more complicates this picture. Thus, strictly speaking, inside of a particle operates the non-linear field theory and apparently the principle of superposition should here not have place.

Chapter 3. The quantum and electromagnetic forms of electron theory

1.0. Introduction

In the chapter 2 we have shown that the Dirac electron equation is the equation of EM wave, moving along a ring trajectory. Thus, the difference between two forms – quantum and electromagnetic - consists only in the mathematical form of record: the complex form of the EM equations corresponds to the operationally-matrix form of the quantum equations.

The Dirac electron theory has a lot of particularities. In the modern interpretation these particularities are considered as mathematical features that do not have a physical meaning. The electromagnetic form of part of them we have considered in the chapter 2. On the basis of the chapter 2 we will show also that all other mathematical particularities of the Dirac electron theory have the known electrodynamics sense.

2.0. Electrodynamics meaning of the forms of the Dirac equations

2.2. The quantum Dirac equation forms with mass

There are two bispinor Dirac equations (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960) (the description of the equation characteristics and parameters see in the chapter 2):

$$\left[(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} mc^2 \right] \psi = 0, \quad (1.1)$$

$$\left[(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} mc^2 \right] \psi = 0, \quad (1.2)$$

which correspond to two signs of the relativistic expression of the electron energy:

$$\varepsilon = \pm \sqrt{c^2 \vec{p}^2 + m^2 c^4}, \quad (1.3)$$

but for each sign of the expression (1.3) there are two Hermitian-conjugate Dirac equations. Thus there are two Hermitian-conjugate equations, corresponding to the minus sign of the expression (1.3):

$$\left[(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} mc^2 \right] \psi = 0, \quad (1.4')$$

$$\psi^+ \left[(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) + \hat{\beta} mc^2 \right] = 0, \quad (1.4'')$$

and two equations that correspond to plus signs of (1.3):

$$\left[(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} mc^2 \right] \psi = 0, \quad (1.5')$$

$$\psi^+ \left[(\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) - \hat{\beta} mc^2 \right] = 0, \quad (1.5'')$$

We will use further the wave function in the matrix form of the plane EM wave, moving as in the chapter 2 along y - axis:

$$\psi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \psi^+ = (E_x \quad E_z \quad -iH_x \quad -iH_z), \quad (1.6)$$

which with the following choice of the Dirac matrices

$$\hat{\alpha}_0 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & \hat{\sigma}_0 \end{pmatrix}, \quad \hat{\alpha} = \begin{pmatrix} 0 & \hat{\sigma} \\ \hat{\sigma} & 0 \end{pmatrix}, \quad \hat{\beta} \equiv \hat{\alpha}_4 = \begin{pmatrix} \hat{\sigma}_0 & 0 \\ 0 & -\hat{\sigma}_0 \end{pmatrix}, \quad (1.7)$$

where $\hat{\sigma}$ are Pauli spin matrices, give the correct electrodynamics expressions.

2.2. The EM Dirac equation forms

Let us consider first two Hermitian-conjugate equations, corresponding to the minus sign of the expression (1.3). Using (1.6), from (1.4') and (1.4'') we obtain:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -\vec{j}_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = \vec{j}_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = -\vec{j}_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = \vec{j}_x^m \end{array} \right. , (1.7'), \quad \left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = \vec{j}_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = -\vec{j}_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = \vec{j}_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = -\vec{j}_x^m \end{array} \right. , (1.7'')$$

where

$$\vec{j}^e = i \frac{\omega_e}{4\pi} \vec{E} = i \frac{1}{4\pi} \frac{c}{r_e} \vec{E}, \quad (1.8')$$

$$\vec{j}^m = i \frac{\omega_e}{4\pi} \vec{H} = i \frac{1}{4\pi} \frac{c}{r_e} \vec{H}, \quad (1.8'')$$

are the complex currents, in which $\omega_e = \frac{2mc^2}{\hbar}$, and $r_e = \frac{\hbar}{2mc}$. Thus, the equations (1.4') and (1.4'') are Maxwell equations with complex currents. As we see, *the Hermitian-conjugate equations (1.7) and (1.8) differ by the current directions.*

Let us consider now the equations that correspond to plus signs of (1.3). The electromagnetic form of the equation (1.5') is:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -\vec{j}_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = \vec{j}_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = -\vec{j}_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = \vec{j}_x^m \end{array} \right. , \quad (1.9)$$

Obviously, the electromagnetic form of the equation (1.5'') will have the opposite signs of the currents comparatively to (1.9).

Comparing (1.9) and (1.7) we can see that the *equation (1.9) can be considered as the Maxwell equation of the retarded wave*. If we don't want to use the retarded wave, we can transform the wave function of the retarded wave to the form:

$$\Psi_{ret} = \begin{pmatrix} E_x \\ -E_z \\ iH_x \\ -iH_z \end{pmatrix}, \quad (1.10)$$

Then, contrary to the system (1.9) we get the system (1.8). The transformation of the function Ψ_{ret} to the function Ψ_{adv} is called the charge conjugation operation.

Note that the electron and positron wave functions can be considered as the retarded and advanced waves. So the above result links also with the theory of advanced waves of Wheeler and Feynman (Wheeler and Feynman, 1945; Wheeler, 1957). (See also Dirac's work on time-symmetric classical electrodynamics (Dirac, 1938), and about this theme - Konopinski's book (Konopinski, 1980).

3.0. Electrodynamics meaning of the bispinor forms

It is known that there are 16 Dirac matrices of 4x4 dimensions. We use the set of matrices which used Dirac himself and we will name it α -set (1.4).

It can be shown that the tensor dimension of bilinear form follows from its nonlinear electrodynamics forms. Enumerate corresponding Dirac's matrices (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955):

$$1) \quad \hat{\alpha}_4 \equiv \hat{\beta}, \quad (3.1')$$

$$2) \quad \hat{\alpha}_\mu = \{\hat{\alpha}_0, \hat{\alpha}\} \equiv \{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4\}, \quad (3.1'')$$

$$3) \quad \hat{\alpha}_5 = \hat{\alpha}_1 \cdot \hat{\alpha}_2 \cdot \hat{\alpha}_3 \cdot \hat{\alpha}_4, \quad (3.1''')$$

$$4) \quad \hat{\alpha}_\mu^A = \hat{\alpha}_5 \cdot \hat{\alpha}_\mu, \quad (3.1''')$$

$$5) \quad \hat{\alpha}_{\mu\nu} = -\hat{\alpha}_{\nu\mu} = \begin{cases} i\hat{\alpha}_\nu \hat{\beta} \hat{\alpha}_\mu, & \mu \neq \nu \\ 0, & \mu = \nu \end{cases}, \quad (3.2)$$

where 1) scalar, 2) 4-vector, 3) pseudoscalar, 4) 4-pseudovector, 5) antisymmetrical tensor of second rank are.

Let's calculate electrodynamics values corresponding to these matrices: $O = \Psi^+ \hat{\alpha} \Psi$, where Ψ is given by (1.6):

1) $\Psi^+ \hat{\alpha}_4 \Psi = (E_x^2 + E_z^2) - (H_x^2 + H_z^2) = \vec{E}^2 - \vec{H}^2 = 8\pi I_1$, where I_1 is the *first scalar* (invariant) of Maxwell theory, i.e. the Lagrangian of electromagnetic field in vacuum;

2) $\Psi^+ \hat{\alpha}_0 \Psi = \vec{E}^2 + \vec{H}^2 = 8\pi U$, where U is the *energy density* of electromagnetic field;

$\Psi^+ \hat{\alpha}_y \Psi = -\frac{8\pi}{c} \vec{S}_{Py} = -8\pi c \vec{g}_y$, where \vec{g}_y is the *momentum density* of the

electromagnetic wave field moved along the Y -axis. As it is known, the value $\left\{ \frac{1}{c} U, \vec{g} \right\}$ is 4-vector of energy-momentum.

3) $\Psi^+ \hat{\alpha}_5 \Psi = 2(E_x H_x + E_z H_z) = 2(\vec{E} \cdot \vec{H})$, which is the *pseudoscalar* of electromagnetic field, and $(\vec{E} \cdot \vec{H})^2 = I_2$ is the second scalar (invariant) of electromagnetic field theory.

4) $\Psi^+ \hat{\alpha}_5 \hat{\alpha}_0 \Psi = 2(E_x H_x + E_z H_z) = 2(\vec{E} \cdot \vec{H})$

$\Psi^+ \hat{\alpha}_5 \hat{\alpha}_1 \Psi = -2i(E_x E_z - H_x H_z)$,

$$\psi^+ \hat{\alpha}_5 \hat{\alpha}_2 \psi = 0,$$

$$\psi^+ \hat{\alpha}_5 \hat{\alpha}_3 \psi = -i(E_x^2 - E_z^2 - H_x^2 + H_z^2).$$

As we will show in the chapter 5, the 4-pseudovector is connected with spirality of particles.

5) Tensor $\psi^+ \hat{\alpha}_{\mu\nu} \psi$ we can write in compact form:

$$(\alpha_{\mu\nu}) = \begin{pmatrix} 0 & E_x^2 - E_z^2 + H_x^2 - H_z^2 & 0 & -2(E_x H_z + E_z H_x) \\ -(E_x^2 - E_z^2 - H_x^2 + H_z^2) & 0 & 2(E_x E_z - H_x H_z) & 0 \\ 0 & -2(E_x E_z - H_x H_z) & 0 & -2(E_x H_x - E_z H_z) \\ 2(E_x H_z + E_z H_x) & 0 & 2(E_x H_x - E_z H_z) & 0 \end{pmatrix}$$

As we will show below this tensor defines the Lorentz force.

4.0. About statistical interpretation of the wave function

As it is known, from the Dirac equation the probability continuity equation can be obtained (Akhiezer and Berestetskii, 1965; Bethe, 1964; Schiff, 1955; Fermi, 1960):

$$\frac{\partial P_{pr}(\vec{r}, t)}{\partial t} + \text{div } \vec{S}_{pr}(\vec{r}, t) = 0, \quad (4.1)$$

Here $P_{pr}(\vec{r}, t) = \psi^+ \hat{\alpha}_0 \psi$ is the probability density, and $\vec{S}_{pr}(\vec{r}, t) = -c \psi^+ \hat{\alpha} \psi$ is the probability flux density. Using the above results we can obtain: $P_{pr}(\vec{r}, t) = 8\pi U$ and $\vec{S}_{pr} = c^2 \vec{g} = 8\pi \vec{S}$. Then the electromagnetic form of the equation (3.15) is:

$$\frac{\partial U}{\partial t} + \text{div } \vec{S} = 0, \quad (4.2)$$

which is the form of *energy-momentum conservation law* of the EM field.

5.0. The electrodynamics meaning of the matrices choice

According to Fermi (Fermi, 1960) "it can prove that all the physical consequences of Dirac's equation do not depend on the special choice of Dirac's matrices... In particular it is possible to interchange the roles of the four matrices by unitary transformation. So, their differences are only apparent".

The matrix sequence $(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3)$ agrees with the electromagnetic wave, which has $-y$ - direction. A question arises: how to describe the waves, which have x and z - directions? Introducing the axes' indexes, which indicate the electromagnetic wave direction, we can write three groups of the matrices, each of which corresponds to one and only one wave direction:

$$(\hat{\alpha}_{1x}, \hat{\alpha}_{2y}, \hat{\alpha}_{3z}), (\hat{\alpha}_{2x}, \hat{\alpha}_{3y}, \hat{\alpha}_{1z}), (\hat{\alpha}_{2z}, \hat{\alpha}_{1y}, \hat{\alpha}_{3x}).$$

Let us choose now the wave function forms, which give the correct Maxwell equations for the x and z - directions. Taking into account (1.6) as the initial form of the $-y$ - direction, from it, by means of the indexes' transposition around the circle we will get other forms.

Since in this case the Poynting vector has the minus sign, we can suppose that the transposition must be counterclockwise. Let us examine this supposition, checking the Poynting vector values:

The sets $(\hat{\alpha}_{1x}, \hat{\alpha}_{2y}, \hat{\alpha}_{3z}), (\hat{\alpha}_{2x}, \hat{\alpha}_{3y}, \hat{\alpha}_{1z}), (\hat{\alpha}_{2z}, \hat{\alpha}_{1y}, \hat{\alpha}_{3x})$ correspond to the wave

$$\text{functions } \psi(y) = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \psi(x) = \begin{pmatrix} E_z \\ E_y \\ iH_z \\ iH_y \end{pmatrix}, \psi(z) = \begin{pmatrix} E_y \\ E_x \\ iH_y \\ iH_x \end{pmatrix}, \text{ and to non-zero Poynting vectors}$$

$$\psi^+ \hat{\alpha}_{2y} \psi = -2[\vec{E} \times \vec{H}]_y, \psi^+ \hat{\alpha}_{2x} \psi = -2[\vec{E} \times \vec{H}]_x, \psi^+ \hat{\alpha}_{2z} \psi = -2[\vec{E} \times \vec{H}]_z \text{ respectively.}$$

As we see, we took the correct result. We can suppose now that by the clockwise indexes' transposition of the wave functions will describe the electromagnetic waves, which move in a positive direction along the co-ordinate axes. Let us prove this:

The sets $(\hat{\alpha}_{1_x}, \hat{\alpha}_{2_y}, \alpha_{3_z})$, $(\hat{\alpha}_{2_x}, \hat{\alpha}_{3_y}, \hat{\alpha}_{1_z})$, $(\hat{\alpha}_{2_z}, \hat{\alpha}_{1_y}, \hat{\alpha}_{3_x})$ correspond to the wave

$$\text{functions: } \psi(y) = \begin{pmatrix} E_z \\ E_x \\ iH_z \\ iH_x \end{pmatrix}, \quad \psi(x) = \begin{pmatrix} E_y \\ E_z \\ iH_y \\ iH_z \end{pmatrix}, \quad \psi(z) = \begin{pmatrix} E_x \\ E_y \\ iH_x \\ iH_y \end{pmatrix}, \text{ and to following non-zero}$$

Poynting vectors $\psi^+ \hat{\alpha}_{2_y} \psi = 2[\vec{E} \times \vec{H}]_y$, $\psi^+ \hat{\alpha}_{2_x} \psi = 2[\vec{E} \times \vec{H}]_x$, $\psi^+ \hat{\alpha}_{2_z} \psi = 2[\vec{E} \times \vec{H}]_z$ respectively. As we see, once again we get the correct results.

Now we will prove that the above choice of the matrices and wave functions gives the correct electromagnetic equation forms. Using for example equation (1.5') and transposing the indexes clockwise we obtain for the positive direction of the electromagnetic wave the following results for x, y, z -directions correspondingly:

$$\begin{cases} \frac{1}{c} \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} = -j_y^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} = j_z^m \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} = -j_z^e \\ \frac{1}{c} \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = j_y^m \end{cases}, \quad \begin{cases} \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial x} = -j_z^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial x} = j_x^m \\ \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial x} = -j_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial x} = j_z^m \end{cases}, \quad \begin{cases} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_y}{\partial x} = -j_x^e \\ \frac{1}{c} \frac{\partial H_y}{\partial t} + \frac{\partial E_x}{\partial x} = j_y^m \\ \frac{1}{c} \frac{\partial E_y}{\partial t} - \frac{\partial H_x}{\partial x} = -j_y^e \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_y}{\partial x} = j_x^m \end{cases}, \quad (5.1)$$

As we can see, we have obtained three equation groups, each of which contains four equations, *as is necessary for the description of all electromagnetic wave directions*. In the same way for all other forms of the Dirac equation analogue results can be obtained.

Obviously, it is also possible via canonical transformations to choose the Dirac matrices in such a way that the electromagnetic wave will have any direction. Let us show it.

5.1. The EM meaning of canonical transformations of Dirac's matrices and bispinors

The choice (1.7) of the matrices is not unique (Akhiezer and Berestetskii, 1965; Schiff, 1955; Fock, 1932). As it is known, there is a free transformation of a kind: $\alpha = S a' S^+$, where S is a unitary matrix, called the canonical transformation operator and also the wave functions ψ' transformation $\psi = S \psi'$, which does not change the results of the theory.

If we choose matrices α' as:

$$\hat{\alpha}'_1 = \begin{pmatrix} \hat{\sigma}_x & 0 \\ 0 & \hat{\sigma}_x \end{pmatrix}, \hat{\alpha}'_2 = \begin{pmatrix} \hat{\sigma}_y & 0 \\ 0 & -\hat{\sigma}_y \end{pmatrix}, \hat{\alpha}'_3 = \begin{pmatrix} \hat{\sigma}_z & 0 \\ 0 & \hat{\sigma}_z \end{pmatrix}, \hat{\alpha}'_4 = \begin{pmatrix} 0 & -i\hat{\sigma}_y \\ i\hat{\sigma}_y & 0 \end{pmatrix}, \quad (5.2)$$

then the functions ψ will be connected to functions ψ' according to the relationships:

$$\psi_1 = \frac{\psi'_1 - \psi'_4}{\sqrt{2}}, \quad \psi_2 = \frac{\psi'_2 + \psi'_3}{\sqrt{2}}, \quad \psi_3 = \frac{\psi'_1 + \psi'_4}{\sqrt{2}}, \quad \psi_4 = \frac{\psi'_2 - \psi'_3}{\sqrt{2}}, \quad (5.3)$$

The unitary matrix S , which corresponds to this transformation, is equal to:

$$S = \frac{1}{\sqrt{2}} \begin{Bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{Bmatrix}, \quad (5.4)$$

It is not difficult to check that by means of this transformation we will also receive the equations of the Maxwell theory. Actually, using (1.6) and (5.3) it is easy to receive:

$$\frac{\psi'_1 - \psi'_4}{\sqrt{2}} = E_x, \quad \frac{\psi'_2 + \psi'_3}{\sqrt{2}} = E_z, \quad \frac{\psi'_1 + \psi'_4}{\sqrt{2}} = iH_x, \quad \frac{\psi'_2 - \psi'_3}{\sqrt{2}} = iH_z, \quad (5.5)$$

whence:

$$\psi' = \frac{\sqrt{2}}{2} \begin{pmatrix} E_x + iH_x \\ E_z + iH_z \\ E_z - iH_z \\ -E_x + iH_x \end{pmatrix} \quad (5.6)$$

Substituting these functions in the Dirac equation we will receive the correct Maxwell equations for the electromagnetic waves in double quantity. It is possible to assume, that the functions ψ' correspond to the electromagnetic wave, moving under the angle of 45 degrees to both coordinate axes.

Thus, from above it follows that *every choice of the Dirac matrices defines only the direction of the initial electromagnetic wave*. Obviously, this is a physical origin why "the physical consequences of Dirac's equation do not depend on the special choice of Dirac's matrices" (Fermi, 1960).

6.0. The electromagnetic form of the EM electron theory Lagrangian

As a Lagrangian of the Dirac theory can take the expression (Schiff, 1955):

$$L_D = \psi^+ (\hat{\epsilon} + c\hat{\alpha} \hat{p} + \hat{\beta} mc^2) \psi, \quad (6.1)$$

For the electromagnetic wave moving along the $-y$ -axis the equation (6.1) can be written:

$$L_D = \frac{1}{c} \psi^+ \frac{\partial \psi}{\partial t} - \psi^+ \hat{\alpha}_y \frac{\partial \psi}{\partial y} - i \frac{mc}{\hbar} \psi^+ \hat{\beta} \psi, \quad (6.2)$$

Transferring each term of (6.2) in the electrodynamics form we obtain the electromagnetic form of the Dirac theory Lagrangian:

$$L_{DM} = \frac{\partial U}{\partial t} + \text{div } \vec{S} - i \frac{\omega}{4\pi} (\vec{E}^2 - \vec{H}^2), \quad (6.3)$$

(Note that in the case of the variation procedure we must distinguish the complex conjugate field vectors \vec{E}^* , \vec{H}^* and \vec{E} , \vec{H}). Using the complex electrical and "magnetic" currents (1.8') and (1.8'') we take:

$$L_{DM} = \frac{\partial U}{\partial t} + \text{div } \vec{S} - (\vec{j}^e \vec{E} - \vec{j}^m \vec{H}), \quad (6.4)$$

It is interesting that since $L_s = 0$ thanks to (1.6), we can take the equation:

$$\frac{\partial U}{\partial t} + \text{div } \vec{S} - (\vec{j}^e \vec{E} - \vec{j}^m \vec{H}) = 0, \quad (6.5)$$

which has the form of the energy-momentum conservation law for the Maxwell equation with current.

7.0. The Lorentz force expression in EM representation

According to our theory for the EM particles stability in the twirled waves (i.e. into the EM particles) the force must appear, which is perpendicular to the trajectory of motion of the EM fields. But in this case the tangential force (by our chose – along the y -axis), must absent, since it would provoke the tangential acceleration of the electron fields.

The expression of Lorentz's force by the energy-momentum tensor of electromagnetic field τ_μ^ν is well known (Tonnelat, 1959; Ivanenko and Sokolov, 1949). This tensor is symmetrical and has the following components:

$$f_\mu = -\frac{1}{4\pi} \frac{\partial \tau_\mu^\nu}{\partial x^\nu} \equiv -\frac{1}{4\pi} \partial_\nu \tau_\mu^\nu, \quad (7.1)$$

Here, first three components describe the Lorentz force density vector, and fourth component corresponds to the energy conservation law.

Using (7.1) it can be written:

$$f_x = f_z = 0, \quad f_y \equiv - \left(\frac{\partial \bar{g}}{\partial t} + \text{grad } U \right) \quad (7.2)$$

$$f_4 = - \left(\frac{1}{c} \frac{\partial U}{\partial t} + c \text{div } \bar{g} \right), \quad (7.3)$$

As we see by using of the symmetrical energy-momentum tensor we don't obtain the needed components of the force since here $f(y) \neq 0$.

The right result can obtain using antisymmetrical spin tensor $\alpha_{\mu\nu}$ (3.2). Then we have:

$$f_\mu = - \frac{1}{4\pi} \frac{\partial \alpha_\mu^\nu}{\partial x^\nu} \equiv - \frac{1}{4\pi} \partial_\nu \alpha_\mu^\nu, \quad (7.4)$$

or:

$$\begin{cases} f_x = - \left(\frac{\partial \alpha_{12}}{\partial x_2} + \frac{\partial \alpha_{14}}{\partial x_4} \right) \\ f_y = 0 \\ f_z = - \left(\frac{\partial \alpha_{32}}{\partial x_2} + \frac{\partial \alpha_{34}}{\partial x_4} \right) \\ f_0 = 0 \end{cases} \quad (7.5)$$

Using (1.6) and (3.2) we obtain of Lorenz's force components:

$$\begin{aligned} 2\pi f_x &= E_x \left(\frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} \right) + H_z \left(\frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} \right) + \\ &+ H_x \left(\frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} \right) + E_z \left(\frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} \right) \\ f_y &= 0, \end{aligned} \quad (7.6)$$

$$\begin{aligned} 2\pi f_z &= E_x \left(\frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} \right) - H_z \left(\frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} \right) + \\ &+ H_x \left(\frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} \right) - E_z \left(\frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} \right) \\ f_4 &= 0, \end{aligned}$$

For the linear photon all the brackets in (7.6) are equal to zero according to Maxwell's equation. It means that appear no forces in linear photon. When photon rolls up around any of the axis, which are perpendicular to the Y-axis, we will get the additional current terms.

If to take that the field vector of type $\vec{F} = \vec{F}(\vec{r})e^{-i\omega t}$ describes geometrically the vectors rotation, we can for the twirled semi-photon write:

$$\frac{\partial E_x}{\partial t} = i\omega E_x + \frac{\partial E_x}{\partial t}, \quad (7.7)$$

$$\frac{\partial E_z}{\partial t} = i\omega E_z + \frac{\partial E_z}{\partial t}, \quad (7.7'')$$

For spinning photon (E_x, H_z), the force components are (the upper left index shows the spinning axis OZ or OX).

$${}^z f_x = 2i \frac{1}{4\pi} \frac{\omega}{c} E_x (E_x + H_z) = 2 \frac{1}{c} j_\tau \cdot (E_x + H_z), \quad (7.8)$$

for spinning photon (E_z, H_x):

$${}^x f_z = -2i \frac{1}{4\pi} \frac{\omega}{c} E_z (E_z - H_x) = -2 \frac{1}{c} j_\tau \cdot (E_z - H_x), \quad (7.9)$$

$$f_y = 0, \quad (7.10)$$

$$f_4 = 0, \quad (7.11)$$

what corresponds to our representations about the dynamics of twirled semi-photon.

8.0. The equation of the ring EM wave field motion

We can suppose that 4-vector-potential of electromagnetic field, multiplied to the electron charge e , $\left\{ e\varphi, \frac{e}{c} \vec{A} \right\}$ is the 4-vector of the energy-momentum of the curvilinear wave field $\left\{ \varepsilon_p, \vec{p}_p \right\}$ (see chapter 2).

Therefore, the well-known analysis of Dirac's electron equation in the external field can be used for the analysis of the equations of the inner twirled photon field by the changes:

$$\frac{e}{c} \vec{A} \rightarrow \vec{p}_p, \quad e\varphi \rightarrow \varepsilon_p, \quad m \rightarrow 0, \quad (8.1)$$

As it is known (Akhiezer and Berestetskii, 1965; Schiff, 1955), the equation of the electron motion in the external field can be found from the next operator equation, having the Poisson brackets

$$\frac{d\hat{O}}{dt} = \frac{\partial \hat{O}}{\partial t} + \frac{1}{i\hbar} (\hat{O}\hat{H} - \hat{H}\hat{O}), \quad (8.2)$$

where \hat{O} is the physical value operator, whose variation we want to find and \hat{H} is the Hamilton operator of Dirac's equation.

The Hamilton's operator of the Dirac equation is equal (Schiff, 1955; Akhiezer and Berestetskii, 1965):

$$\hat{H} = -c\hat{\alpha} \hat{P} - \hat{\beta} mc^2 + \varepsilon, \quad (8.3)$$

where $\hat{P} = \hat{p} - \vec{p}_p$ is full momentum of twirled photon.

For $\hat{O} = \hat{P}$ from (8.3) we have:

$$\frac{d\hat{P}}{dt} = \left[-grad (e\varphi) - \frac{e}{c} \frac{\partial \vec{A}}{\partial t} \right] + \frac{e}{c} [\vec{v} \times rot \vec{A}], \quad (8.4)$$

or, substitute $\vec{v} = c\hat{\alpha}$, where \vec{v} - velocity of the electron matter, we obtain:

$$\frac{d\vec{P}}{dt} = e\vec{E} + \frac{e}{c} [\vec{v} \times \vec{H}] = f_L, \quad (8.5)$$

Since for the motionless electron $\frac{d\hat{P}_p}{dt} = 0$, the motion equation is:

$$\left(\frac{\partial \vec{p}_p}{\partial t} + grad \varepsilon_p \right) - [\vec{v} \times rot \vec{p}_p] = 0, \quad (8.6)$$

Passing to the energy and momentum densities

$$\vec{g}_p = \frac{1}{\Delta\tau} \vec{p}_p, \quad U_p = \frac{1}{\Delta\tau} \varepsilon_p, \quad (8.7)$$

we obtain the equation of matter motion of twirled photon:

$$\left(\frac{\partial \vec{g}_p}{\partial t} + \text{grad } U_p \right) - [\vec{v} \times \text{rot } \vec{g}_p] = 0, \quad (8.8)$$

Let us analyse the physical meaning of (8.8), considering the motion equation of ideal liquid in form of Lamb's-Gromek's equation (Lamb, 1931). In this case, when the external forces are absent, this equation is:

$$\left(\frac{\partial \vec{g}_l}{\partial t} + \text{grad } U_l \right) - [\vec{v} \times \text{rot } \vec{g}_l] = 0, \quad (8.9)$$

where U_l, \vec{g}_l -energy and momentum density of ideal liquid.

Comparing (8.8) and (8.9) is not difficult to see their mathematical identity. From this follows the interesting conclusion: the inner particles' equation may be interpreted as the motion equation of ideal liquid.

According to (8.5, 8.6) from (8.9) we have

$$\frac{\partial \vec{g}_p}{\partial t} + \text{grad } U_p = \vec{f}_L, \quad (8.10)$$

where f_L is the Lorenz force. As it is known the term $[\vec{v} \times \text{rot } \vec{g}_p]$ in (8.9) is responsible for centripetal acceleration. Probably, we have the same in (8.8). If the "photon' liquid" move along the ring of r_p radius, then the angular motion velocity ω is tied with $\text{rot } \vec{v}$ by expression:

$$\text{rot } \vec{v} = 2\vec{\omega}_p = 2\omega_p \vec{e}_z, \quad (8.11)$$

and centripetal acceleration is

$$\vec{a}_n = \frac{1}{2} \vec{v} \times \text{rot } \vec{v} = \frac{v^2}{r_p} \vec{e}_r = c\omega_p \vec{e}_r, \quad (8.12)$$

where \vec{e}_r is unit radius-vector, \vec{e}_z - is unit vector of OZ -axis. As a result the equation (5.25) has the form of Newton's law:

$$\rho \vec{a}_n = \vec{f}_L, \quad (8.13)$$

This result can be seen as the electromagnetic representation of the Erenfest theorem (Shiff,1955).

Conclusion

The above results proof that the non-linear EM representation of the Dirac theory give the classical explanations of all particularities of the Dirac electron theory, which nevertheless don't contradict to the quantum interpretation.

Chapter 4. Point and non-point solutions of electron equations

1.0. Introduction

1.1. Statement of the problem

The theory of calculation of charge, mass and other characteristics of electron on the basis of the field equations has arisen originally in classical electrodynamics and was developed by W. Kelvin, J. Larmor, H. Lorentz, M. Abraham, A. Poincare, etc. (Pauli, 1958; Ivanenko and Sokolov, 1949). It is based on hypotheses of the field mass and field charge, according to which the particles' own energy or mass is obliged to energy of fields, and the charge of particles is defined by the particles' own fields. These ideas afterwards were transferred to quantum mechanics. But neither classical, nor quantum theories could explain consistently the nature of mass and charge of elementary particles, although for the electron some consecutive theories have been constructed.

1.2. The general requirements to the classical electron mass theory

At first we will address to the hypothesis of electron field mass within the framework of classical electrodynamics (Lorentz, 1916; Ivanenko and Sokolov, 1949).

According to the hypothesis, which has been put forward in the end of the 19th century by J.J. Thomson and advanced by H. Lorentz, M. Abraham, A. Poincare, etc., the electron's own energy (or its mass) is completely caused by the energy of the electromagnetic field of electron. In the same way it is supposed that the electron momentum is obliged to the momentum of the field. Since electron, as any mechanical particle, possesses the momentum and energy, which are together the 4-vector of the generalized momentum, the necessary condition of success of the theory will be the proof that the generalized momentum of an electromagnetic field is a 4-vector.

Thus, for the success of the field mass theory the following conditions should be satisfied at least:

At first, it is necessary to receive final value of the field energy, generated by a particle, which could be precisely equated to final energy of a particle (i.e. product of the mass by the square of the light speed).

At second, the value of a momentum of the field, generated by a particle, must not only be final, but also has the proper correlation with energy, forming with the last a four-dimensional vector.

Thirdly, the theory should manage to deduce the equation of movement of electron.

Fourthly, it is necessary to obtain of electron spin, as a spin of a field (that needs the quantum generalization of the theory of field mass, since a spin is quantum effect).

The analysis shows, that there are two conditions, by which the generalized field momentum G_μ is a 4-vector.

In case of space without charges the size

$$G_\mu = \frac{i}{c} \int T_{\mu 4}(dr), \quad (1.1)$$

will represent a 4-vector if divergence of energy tensor of a field turns into zero:

$$\frac{\partial T_{\mu\lambda}}{\partial x_\lambda} = 0, \quad (1.2)$$

For example, the electromagnetic field, which is located in a space without charges, satisfies similar conditions. In particular, due to this fact, in the photon theory, EM field is characterized not only by energy, but also by momentum.

2) The condition, by which the energy and momentum of an electromagnetic field form a 4-vector at the presence of charges, is formulated by the Laue theorem. According to the last, at the presence of charges the size G_μ is a 4-vector only in the case when in the coordinate system, relatively to which electron is in rest, for all the energy tensor components the following parity is observed

$$\int T_{\mu\nu}^0(d\vec{r}_o) = 0, \quad (1.3)$$

except for the component T_{44}^0 , the integral of which is a constant and is equal to full energy of the field, generated by particle (here $(d\vec{r}_o)$ is elementary volume in reference system, in which the electron is in rest). The equality (1.3) expresses a necessary condition, by which the whole particle charge should be in balance.

We can equate this field energy to the particle's own energy, expressing in this way the basic idea of a field hypothesis. According to the last:

$$m_e = \frac{\mathcal{E}_e}{c^2} = \frac{1}{c^2} \int T_{44}^0(d\vec{r}_o), \quad (1.4)$$

Thus, the mass of a particle from the field point of view can be defined in two ways:

1) proceeding from EM momentum of a field G_1 it is possible to define mass as factor of proportionality between a field momentum and three-dimensional speed of a particle.

2) if we consider the electron's own energy as equal or conterminous to the energy of a field, and mass as the ratio of a field energy $\frac{c}{i} G_4$, to a square of light speed (i.e. as the fourth component of a generalized momentum).

The attempts to execute this program, proceeding from classical linear Maxwell theory, have led to difficulties. In particular, it was not possible to prove the Laue theorem (Tonnelat, 1959). In the classical theory the dynamics (mechanics) and electrodynamics are completely independent from each other. Electromagnetic actions are characterized by component T_0^0 of an energy-momentum tensor of

an electromagnetic field. It does not include the energy and momentum of the substance, which should be subsequently inserted. The attempts of Lorentz and Poincare to coordinate the theory on the basis of the assumption that energy of substance has an electromagnetic origin, have not led to a positive result. In Lorentz electron theory (linear in essence) existence of charges it is possible to explain only by introduction of forces of non-electromagnetic origin.

Nevertheless (Sokolov and Ivanenko, 1949), there were also a number of successes, which carried a hope to solve this problem by any change of the theory. The most perspective change of Maxwell-Lorentz theory appeared to be its non-linear generalization.

1.3. Non-linear classical electrodynamics

In the chapter 2 within the framework of CWED we have received the non-linear equation for the electromagnetic (EM) electron and have shown that on sufficient distance from a particle it coincides with the linear Dirac electron equation. But unfortunately the solution of the non-linear equation of the curvilinear electromagnetic wave is not received yet. Its first approach – the non-linear Heisenberg equation - also did not manage to be solved (although here the encouraging results have been received).

We will show the known classical non-linear theories of Gustav Mie, M. Born - L. Infeld, E. Schroedinger etc. represent the approximate solutions of non-linear equation of CWED, which enable us to estimate the sizes of a particle and distribution of a field in approach of spherical electron. Besides, the nonlinear theories find out an opportunity of description of EM electron as point or not point, depending on the used mathematics.

2.0. Gustav Mie approach to the electron theory

2.1. Prior history

Gustav Mie made the first attempt to construct a purely electromagnetic theory of charged particles. (Mie, 1912a; 1912b; 1913; Pauli, 1958; Tonnelat, 1959). Proceeding from some formally irreproachable hypothetical non-linear generalization of electrodynamics, he managed to construct a theory, which has overcome all difficulties of the classical theory.

As we have said above, in the theory of the electron before G. Mie (Bialynicki-Birula, 1983), the electron was not treated as a purely electromagnetic entity, but it was also made of other stuff, like, for example, Poincare stresses and the mechanical mass. Mie wanted only the electromagnetic field to be responsible for all the properties of the electron. In particular, he wanted the electromagnetic current to be made of electromagnetism.

In order to achieve this goal, Mie assumed that the potential four-vector enters directly into the Lagrangian and not only through the field strength. The generation of the current has been achieved in this manner, but the price was very high. The potentials acquired a physical meaning and the gauge invariance was lost. This property has been found unacceptable by other physicists and the theory of Mie has been shelved for many long years.

2.2. G. Mie theory

In his theory Mie has made two essential steps (Pauli, 1958; Tonnelat, 1959). At first, Mie was the first who suggested in the construction of the theory leaning on a Lagrangian, dependent on fundamental invariants. At second, to get rid of Poincare-Lorentz forces that have non-electromagnetic origin, Mie entered a uniform sight at a field and substance. He set a problem in order to generalize the field equations and the energy-momentum tensor of Maxwell-Lorentz theory in a way that inside the elementary charged particles the repulsion Coulomb forces would counterbalanced by other forces of E origin also, and outside of particles the deviation from ordinary electrodynamics would imperceptible. He assumed that any energy and substance has an electromagnetic origin, and sets as the purpose to deduce properties and characteristics of charges from properties of a field.

About the kind of Lagrangian L , which is frequently called a world function, in non-linear electrodynamics it is possible to make some general statements. The independent invariants of an electromagnetic field, which can be formed from bivector $F_{\mu\nu}$ (where $F_{\mu\nu}$ are the tensor components of electromagnetic field strengths) and a vector $A_\mu = (i\varphi, \vec{A}) = (i\varphi, A_i) = (A_4, A_i)$ are the following:

- 1) The square of bivector $F_{\mu\nu}$: $I_1 = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$; 2) the square of a pseudo-vector $I_2 = \frac{1}{4} F_{\mu\nu} F^{*\mu\nu}$ (where $F^{*\mu\nu}$ is the dual electromagnetic tensor).
- 3) The square of a 4-vector of

electromagnetic potential A_μ : $I_3 = A_\mu A^\mu$; 4) The square of a vector: $I_4 = F_{\mu\nu} A_\mu$; 5) The square of a vector: $I_5 = F_{\mu\nu}^* A_\mu$.

Therefore L can depend only on these five invariants. If L is equal to the first of the specified invariants, the field equations are degenerated into ordinary equations of the electron theory for space without charges. Thus, L can noticeably differ from $\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ only inside the material particles.

Invariant 2 can be included in L only as a square, in order not to break the invariance, concerning spatial reflections. Invariants 3-5 break the gauge invariance. Further statements about the world function L cannot be made. Thus, for the selection of L there are an infinite number of opportunities.

Gustav Mie accepted as initial the following Lagrangian:

$$L_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(\pm \sqrt{A_\mu A^\mu}\right), \quad (2.1')$$

or

$$L_{Mi} = \frac{1}{8\pi} (E^2 - H^2) - f\left(\pm \sqrt{A_\mu A^\mu}\right), \quad (2.1'')$$

where f is some function.

Using this Lagrangian (Tonnelat, 1959), Gustav Mie managed to receive the final energy (or mass) of the charged particle as a value completely caused by the energy of the field of this particle. Besides, in this theory the Laue theorem of stability is carried out and the proper correlation between energy and momentum of a particle is reached.

For further analysis it is also useful to mention the attempt of H. Weyl (Pauli, 1958) to interpret on the basis of Mie theory the asymmetry (with respect to distinction of masses) of both sorts of electricity.

If L is not a rational function of $\sqrt{A_\mu A^\mu}$, it is possible to put:

$$L^+_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(+ \sqrt{A_\mu A^\mu}\right), \quad (2.2')$$

$$L^-_{Mi} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - f\left(- \sqrt{A_\mu A^\mu}\right), \quad (2.2'')$$

Thus, if L is a multiple-valued function of the invariants mentioned above, it is obviously possible to choose as world functions for positive and negative charges various unequivocal branches of this function.

2.3. Connection the Mie theory with the CWED

Let's show, that Mie Lagrangian after some additions can be submitted as Lagrangian similar to Lagrangian of CWED (and consequently, of QED).

As we know (Pauli, 1958; Sommerfeld, 1958), the charge density is not invariant concerning Lorentz transformation, but a charge is. Also it is known, that the square of 4-potential, i.e. $I_3 = A_\mu A^\mu$, is invariant concerning Lorentz transformation, but it is not invariant relatively to gauge transformations. But it appears that the product of a square of a charge on I_3 will be an invariant concerning both Lorentz and gauge transformations. Let's show this.

2.3.1. Larmor - Schwarzschild invariant

According to (Pauli, 1958) and (Sommerfeld, 1958), R. Schwarzschild (Schwarzschild, 1903), entered the value

$$S_w = \varphi - \frac{\vec{v}}{c} \cdot \vec{A}, \quad (2.3)$$

which he called "electrokinetic potential", and has shown, that this value, being multiplied by density of a charge, forms the relativistic invariant;

$$L' = \rho S_w = \rho \left(\varphi - \frac{\vec{v}}{c} \cdot \vec{A} \right) = -\frac{1}{c} j_\mu \cdot A^\mu, \quad (2.4)$$

where $j_\mu = \{ic\rho, \rho\vec{v}\}$ is 4-current density, $A^\mu = \{\varphi, \vec{A}\}$ is 4-potential. Using (2.4) Schwarzschild has formed the following Lagrange function:

$$L = \frac{1}{2} \int (H^2 - E^2) dV + \int \rho \left(\varphi - \frac{\vec{v} \cdot \vec{A}}{c} \right) dV, \quad (2.5)$$

and by time-integration (2.5) he has received the function of action.

Thus, in 4-dimensional designations the Schwarzschild Lagrange function density (or Lagrangian) will be written down as follows:

$$\bar{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} j_\mu A^\mu, \quad (2.6)$$

and the Lagrange function will be:

$$L = \frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d\tau - \frac{1}{c} \int j_\mu A^\mu d\tau, \quad (2.7)$$

(In the note 10 to the book (Pauli, 1958) Pauli marked that before Schwarzschild the same Lagrangian has been suggested by J.J. Larmor (Larmor, 1900)).

Let's consider now the radicand in the Ми Lagrangian:

$$A_\mu^2 \equiv A_\mu A_\mu = -\varphi^2 + A_i^2, \quad (2.8)$$

Multiplying it on the squares of density of a charge and a square of a charge, we shall receive accordingly:

$$e^2 A_\mu^2 = -(e\varphi)^2 + (e\vec{A})^2, \quad (2.9)$$

We will enter the values of density of energy of interaction and energy of electron interaction, accordingly:

$$U_e = \rho\varphi, \quad \varepsilon_e = e\varphi, \quad (2.10)$$

and also the density of momentum and the momentum of electron interaction, accordingly:

$$g_{ei} = \frac{1}{c} \rho A_i, \quad p_{ei} = \frac{1}{c} e A_i, \quad (2.11)$$

Then from (2.9) we shall receive:

$$e^2 A_\mu^2 = -\varepsilon_e^2 + (c\vec{p}_e)^2, \quad (2.12)$$

Since $(\hat{\alpha}_0 \varepsilon)^2 = \varepsilon^2$, $(\hat{\alpha} \vec{p})^2 = \vec{p}^2$ take place, these expressions can be also written down as:

$$e^2 A_\mu^2 = -(\varepsilon_e^2 - c^2 p_{ei}^2) = -\left((\hat{\alpha}_0 \varepsilon_e)^2 - c^2 (\hat{\alpha} \vec{p}_e)^2 \right), \quad (2.13)$$

Using the above-stated results, for non-linear part of the Mie Lagrangian $L_{Mie}^N = f(\pm \sqrt{A_\mu A^\mu})$, we can accept the expression:

$$f(\pm \sqrt{A_\mu A^\mu}) = e(\pm \sqrt{\varphi^2 - c^2 \vec{A}^2}), \quad (2.14)$$

Using of Dirac matrixes it is easy to receive the following decomposition:

$$\sqrt{e^2 A_\mu^2} = \mp (\hat{\alpha}_0 \varepsilon_e \pm c \hat{\alpha} \vec{p}_e), \quad (2.15)$$

that gives for non-linear part of Lagrangian the expression:

$$L_{Mie}^{Ne} = \mp (\hat{\alpha}_0 \varepsilon_e \pm c \hat{\alpha} \vec{p}_e), \quad (2.16)$$

Taking into account that

$$\hat{\beta} mc^2 = -(\varepsilon_p - c \hat{\alpha} \cdot \vec{p}_p), \quad (2.17)$$

we see that we can enter in the Lagrangian the mass term of the Dirac equation. Thus, it is possible to assert that Mie Lagrangian can be transformed to have the form of the Lagrangian of the non-linear field theory, corresponding to the theory of the electron of CWED and QED also.

The use of these expressions leads to the Dirac equations of electron and positron, and gives the basis to the Weyl's attempt to interpret the asymmetry of both sorts of electricity not in connection with mass, but in connection with distinction particle - antiparticle.

Thus, the assumption of Mie that internal properties of electron are described by an electromagnetic field, corresponds to the results of CWED. Actually in the chapter 2 we result that the energy, mass and charge of particle are defined by the inner potentials of this particle. If to accept that potentials inside a particle correspond to an energy-momentum of the particle field, it makes the potentials the physically certain values, which however are not measurable outside of a particle. In other words, the potentials are here the *hidden parameters* of elementary particles.

Do these results contradict to the experimental results of modern physics?

As it is known in classical electrodynamics the potentials play the role of the mathematical auxiliary values and have no physical sense. But as it appears, in framework of quantum mechanics the potentials have physical sense that is proved by Aharonov-Bohm experiment (Aharonov and Bohm, 1959; Feynman, Leighton and Sands, 1989).

As an example of calculation of electron parameters in framework of classical theory, we will consider the results of the Born - Infeld theory (Born and Infeld, 1934) and show, that these results can be considered as some approximation of CWED solution.

3.0. Born-Infeld nonlinear theory

M. Born and L. Infeld revived Mie's theory and proposed a specific model. The Born-Infeld theory (Born and Infeld, 1934) rests on the simplest possible Lagrangian: the square root of the determinant of a second rank covariant tensor. Such a structure automatically guarantees the invariance of the theory under arbitrary coordinate transformations, making the fully relativistic and gauge invariant non-linear electrodynamics.

M. Born and L. Infeld proceeded from the idea of a limited value of the electromagnetic field strength of the electron (which is identical to idea of a limited size of the electron as it is shown below). These reasons led them to the following Lagrangian of the non-linear electrodynamics in the vacuum:

$$L_{BI} = \frac{E_0^2}{4\pi} \left(1 - \sqrt{1 - \frac{E^2 - H^2}{E_0^2} - \frac{(\vec{E} \cdot \vec{H})^2}{E_0^4}} \right), \quad (3.1)$$

where E_0 is the maximum field of electron.

We will consider the most important case of the electrostatic field of the point (spherical symmetric) electron. Putting $\vec{H} = 0$, $\vec{E} = -grad\phi$, $\rho(\vec{x} - \xi) = \delta(\vec{r})\delta(t - s)$, we will find according to (3.1):

$$L_n = \frac{E_0^2}{4\pi} \left(1 - \sqrt{1 - \frac{E_r^2}{E_0^2}} \right) - e\phi\delta(\vec{r})$$

Then by the help of the variation principle we obtain:

$$-\frac{1}{4\pi} \frac{\partial D_r}{\partial E_r} - \frac{\partial L}{\partial \phi} = 0$$

where \vec{D} is the electrical induction vector (D-field):

$$D_r = 4\pi \frac{\partial L}{\partial E_r} = \frac{E_r}{\sqrt{1 - \frac{E_r^2}{E_0^2}}}$$

which corresponds to the equation:

$$div\vec{D}_r = 4\pi e\delta(\vec{r})$$

solution of which is:

$$D_r = \frac{e\vec{r}}{r^3}, \quad (3.2)$$

As we see, from point of view of the D-field, the electron should be considered as point particle.

For the electric field (E-field) we obtain:

$$\vec{E}_r = \frac{\vec{D}_r}{\sqrt{1 + \frac{D_r^2}{E_0^2}}} = \frac{e\vec{r}}{r\sqrt{r^4 + r_0^4}}, \quad (3.3)$$

where $r_0 = \sqrt{\frac{e}{E_0}}$ characterize the electron size. In this case, i.e. from point of view of the electric field

(E-field), the electron is not a point particle. This is very important specificity of the non-linear theory in comparison with the linear theory, which can explain, why experiments on scattering of electron can be interpreted so that the electron looks as a point particle (while the renormalization procedure allows to eliminate the infinities).

From above the electron charge density distribution can be found:

$$\rho = \frac{\text{div}E}{4\pi} = \frac{er_0^4}{2\pi r(r^4 + r_0^4)^{3/2}}, \quad (3.4)$$

Thus, in respect to the electric field the electron charge can be considered as distributed mainly in volume of radius r_0 , since by $r \gg r_0$ the density will quickly aspire to zero. Therefore the size r_0 can be considered as effective radius of electron.

Using known values for mass and charge of electron and speed of light, it is possible to receive $r_0 = 2,28 \cdot 10^{-13}$ cm, which is practically equal to classical radius of electron.

Also it is easy to find value for the maximal field of electron, being a field in the center of the electron (at $r = 0$):

$$E_0 = \frac{e}{r_0^2} = 9,18 \cdot 10^{15} \text{ CGS} = 2,75 \cdot 10^{20} \frac{V}{m}.$$

As it is known, the two types of fields and the two definitions of the charge density, corresponding to them, are also described by the theory of the dielectrics. The value:

$$\varepsilon = \frac{D}{E} = \sqrt{\frac{r^4 + r_0^4}{r^4}}, \quad (3.5)$$

which is here a function of the position, can be considered as a "dielectric permeability of electron". On large distances from a charge, when $\frac{r_0}{r} \rightarrow 0$, ε acquires a value equal to unit as in usual

electrodynamics. It is possible to tell that instead of the expression of energy $\frac{e^2}{r^2}$ Born and Infeld take

$\frac{e^2}{\varepsilon r^2}$, and then the reduction of r is compensated by increase of ε so the full energy remains as

final. (It is possible to assume, that the presence of physical vacuum should make the amendment to value of dielectric permeability, and at the same time, to values of potential of electron, its size and other characteristics).

Thus, proceeding from some formal hypothetical non-linear generalization of electrodynamics, it appeared possible (Ivanenko and Sokolov, 1949):

1. to prove the theorem of stability, i.e. to prove, that in the non-linear theory the electron is stable without introduction of forces of non-electromagnetic origin;

2. to receive the final energy (mass) of electron;

3. to receive the final size of its electric charge;

4. to receive the final size of its electromagnetic field.

3.1. Other Lagrangians of nonlinear theories

Also others Lagrangians have been offered for reception of the non-linear theory.

So E. Schroedinger used the following arbitrary combination for Lagrangian:

$$L_{Sch} = \frac{E_0^2}{8\pi} \ln \left(1 + \frac{E^2 - H^2}{E_0^2} \right), \quad (3.6)$$

It was noted (Ivanenko and Sokolov, 1949), that various variants of formal non-linear electrodynamics lead to close values of coefficients, if to take into account, that the electron radius is equal to classical radius of electron. It was also noted, that the basic defect of these theories, as well as of Mie theory, was the arbitrary choice of Lagrangian, which had no connection with the quantum theory, in particular, with Dirac theory, and did not take into account properties of electron, revealed by the last.

We will show that these theories can be considered as approach of the CWED and that they are mathematically connected to the Dirac electron theory.

4.0. The Born-Infeld theory as an approximation of CWED

Since in general case the CWED (see chapter 2), is the non-linear EM theory, therefore its Lagrangian can contain all possible terms with the invariants of electromagnetic theory. Taking into account the gauge invariance the CWED Lagrangian can be written as some function of the following field invariants:

$$\bar{L} = f_L(I_1, I_2), \quad (4.1)$$

where $I_1 = (\vec{E}^2 - \vec{H}^2)$, $I_2 = (\vec{E} \cdot \vec{H})$ are the invariants of electromagnetic field theory.

Apparently, for each problem the function f_L has its special form, which is unknown before. We can suppose that in all cases there is an expansion of the function f_L in the Taylor – MacLaurent power series with some unknown expansion coefficients. It is also obviously that for the most types of the functions $f_L(I_1, I_2)$ the expansion contains approximately the same set of the terms, which are distinguished only by the constant coefficients, any of which can be equal to zero (as an example of such expansion it is possible to point out the expansion of the quantum electrodynamics Lagrangian for particle at the presence of physical vacuum (Akhiezer and Berestetskii, 1965; Weisskopf, 1936; Schwinger, 1951). Generally this expansion looks like:

$$L_M = \frac{1}{8\pi} (\vec{E}^2 - \vec{B}^2) + L', \quad (4.2)$$

where

$$L' = \alpha (\vec{E}^2 - \vec{B}^2)^2 + \beta (\vec{E} \cdot \vec{B})^2 + \gamma (\vec{E}^2 - \vec{B}^2)(\vec{E} \cdot \vec{B}) + \xi (\vec{E}^2 - \vec{B}^2)^3 + \zeta (\vec{E}^2 - \vec{B}^2)(\vec{E} \cdot \vec{B})^2 + \dots, \quad (4.3)$$

is a part, which is responsible for the non-linear interaction (here $\alpha, \beta, \gamma, \xi, \zeta, \dots$ are constants).

The Lagrangian of the Born-Infeld non-linear electrodynamics can be also expanded into the small parameters $a^2 E^2 \ll 1$ and $a^2 B^2 \ll 1$, where $a^2 = 1/E_0^2$ so that

$$L_{BI} = -\frac{1}{8\pi} (\vec{E}^2 - \vec{B}^2) + \frac{a^2}{32\pi} \left[(\vec{E}^2 - \vec{B}^2)^2 + 4(\vec{E} \cdot \vec{B})^2 \right] + \sum O(\vec{E}^2, \vec{H}^2), \quad (4.4)$$

where $\sum O(\vec{E}^2, \vec{H}^2)$ is the series rest with the terms, containing vectors in powers, which are higher than four. Obviously, under conditions $a^2 E^2 \ll 1$ and $a^2 B^2 \ll 1$ on large distance from the center of a particle (where there is a maximal field) the terms of these series really quickly converge, but on small distance from the center it is, apparently, incorrect and here it needs to take into account the terms of higher degrees.

In the chapter 2 we have shown, that at the first approximation Lagrangian of CWED in electromagnetic form can be represented as following:

$$L_N = -\frac{1}{8\pi} (\vec{E}^2 - \vec{B}^2) + b \left[(\vec{E}^2 - \vec{B}^2) + 4(\vec{E} \cdot \vec{B})^2 \right], \quad (4.5)$$

where b is some constant. Taking into account (4.4), we can write:

$$L_N \approx L_{BI}, \quad (4.6)$$

and receive in the framework of CWED for EM electron the approach solution, like the solution of Born - Infeld theory, stated briefly above.

For this reasons it can similarly show that the CWED Lagrangian approximately coincides with Lagrangian of Schroedinger and others offered Lagrangians of non-linear theories, allowing us to calculate the corresponding characteristics of electron.

Thus, it is not difficult to answer why "various, from the physical point of view, variants of formal non-linear electrodynamics lead to close values of coefficients": as expansion of non-linear Lagrangian (4.3) shows, all of them are approximately equal among themselves and consequently yield close results.

At the same time, since Lagrangian and equations of CWED completely coincide with Lagrangian and the equations of quantum electrodynamics, the Mie theory and its variant – the Born - Infeld theory, is closely connected with the Dirac theory.

Chapter 5. The massive neutrino theory

1.0. Introduction. Neutrino of Standard Model theory

The present status of the problem of neutrino theory is summarized in (Bilenky, Giunti and Kim, 2000; Glashow, 1961; Weinberg, 1967; Salam, 1969). Namely the theory of electroweak interactions including neutrinos combined with the Quantum Chromo-Dynamics (QCD) is now called the Standard Model (SM).

1.1. Neutrinos features

In the Standard Model neutrinos are strictly massless, $m = 0$; all neutrinos are left-handed, with helicity -1, and all antineutrinos are right-handed, with helicity +1; lepton family number is strictly conserved.

But modern experimental evidence indicates that all of these statements are in fact doubtful (Bilenky, Giunti and Kim, 2000).

1.1.1. Helicity and Chirality

In the neutrino theory the conceptions of helicity and chirality play the important role. In the Standard Model theory, neutrino and antineutrino have opposite helicity. It is mathematically possible that this is in fact the *only* difference between neutrinos and antineutrinos, i.e. a right-handed "neutrino" would be an antineutrino. Particles of this sort are called Majorana particles.

As long as the neutrino is massless, its helicity is completely defined, and a Majorana neutrino would be a different particle from its antineutrino. But if neutrinos have mass, and therefore do not travel at exactly the speed of light, it is possible to define a reference frame in which the helicity would be flipped. This means that there is effectively a mixing between the neutrino and the antineutrino (violating lepton number conservation).

Helicity refers to the relation between a particle's spin and direction of motion. To a particle in motion is associated the axis defined by its momentum, and its helicity is defined by the projection of the particle's spin \vec{s} on this axis: the helicity is the component of angular momentum along the

momentum: $h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}| |\vec{p}|}$.

Thus the helicity operator projects out two physical states, with the spin along or opposite the direction of motion - whether the particle is massive or not. If the spin is projected parallel on the direction of motion, the particle is of right helicity, if the projection is antiparallel to the direction of motion, the particle has left helicity.

Something is *chiral* when it cannot be superimposed on its mirror image, like for example our hands. Like our hands, the chiral objects are also classified into left-chiral and right-chiral objects.

In the Standard Model theory the massless neutrino is described by the Dirac lepton equation without the mass term:

$$\alpha^\mu \partial_\mu \psi = 0, \quad \mu = 1,2,3,4, \quad (1.1)$$

which is also satisfied by $\alpha_5 \psi$:

$$\alpha^\mu \partial_\mu (\alpha_5 \psi) = 0, \quad (1.2)$$

where the combination of the α matrices, $\alpha_5 = \alpha_0 \alpha_1 \alpha_2 \alpha_3$ has the properties $\alpha_5^2 = 1$ and $\{\alpha_5, \alpha_\mu\} = 0$. This allows us to define the *chirality operators* which project out left-handed and right-handed states:

$$\psi_L = \frac{1}{2}(1 - \alpha_5) \psi \quad \text{and} \quad \psi_R = \frac{1}{2}(1 + \alpha_5) \psi, \quad (1.3)$$

where ψ_L and ψ_R satisfy the equations $\alpha_5 \psi_L = -\psi_L$ and $\alpha_5 \psi_R = \psi_R$, so the chiral fields are eigenfields of α_5 , regardless of their mass.

We can express any fermion as $\psi = \psi_L + \psi_R$, so that a massive particle always has a *L*-handed as well as a *R*-handed component. In the massless case ψ however "disintegrates" into separate helicity states: the Dirac equation splits into two independent parts, reformulated as the Weyl equations

$$\frac{\hat{\hat{\sigma}} \cdot \hat{\hat{p}}}{|\hat{\hat{\sigma}} \parallel \hat{\hat{p}}|} \left[\frac{1}{2}(1 \pm \alpha_5) \psi \right] = \pm \frac{1}{2}(1 \pm \alpha_5) \psi, \quad (1.4)$$

where $\frac{\hat{\hat{\sigma}} \cdot \hat{\hat{p}}}{|\hat{\hat{\sigma}} \parallel \hat{\hat{p}}|}$ is the helicity operator expressed in terms of the Pauli matrices $\hat{\hat{\sigma}}$.

The **Weyl fermions**, i.e. the massless chiral states $\frac{1}{2}(1 \pm \alpha_5) \psi$, are physical since they correspond to eigenstates of the helicity operator. A massless particle, which is in perpetual motion, thus has an unchangeable helicity. The reason is that its momentum cannot be altered, and its spin of course remains unchanged.

1.1.2. Electromagnetic characteristics of neutrino

It is interesting, that, in spite of neutrality, neutrino possesses electromagnetic characteristics. The analysis of these characteristics (Ternov, 2000) helps us to understand the nature of neutrino mass. Electromagnetic properties of Dirac's and Majorano's neutrino appear to be essentially various. Dirac's massive neutrino as a result of the account of interaction with vacuum receives the magnetic moment. And, the neutrino magnetic moment is directed lengthways a spin, and the magnetic moment antineutrino - against a spin. Thus, the particle and the antiparticle differ by the direction of the magnetic moment. For massive Majorano neutrino, identical to its antiparticle, it appears, that it cannot have neither magnetic nor electric moment.

It also appeared, that the mass and the magnetic moment of neutrino are complex nonlinear functions of field strength and energy of a particle.

Moving in an external field, alongside with the magnetic moment, the Dirac neutrino gets as well the dipole electric moment d_v . Calculations show, that the electric moment of massive Dirac's neutrino, moving in a constant external general view field, is proportional to a pseudo-scalar $(\vec{E} \cdot \vec{H})$, which changes sign at the reflection of time. Thus, the electric moment is induced by an external field, if for this field, the pseudo-scalar $(\vec{E} \cdot \vec{H}) \neq 0$ and its existence does not contradict to T-invariancy of Standard Model. In other words the dipole electric moment of Dirac's neutrino, as well as the magnetic, has dynamic nature.

Note also that there is one electromagnetic characteristic of Dirac's neutrino, which takes place also for Majorano's neutrino: the anapole (or toroidal dipole) moment.

Below we will show that in the framework of the CWED the massive neutrino has the conserved inner (poloidal) helicity, owing to which the above features occur, and is fully described by the Dirac leptonequation.

2.0. Neutrino of CWED

In the previous chapters 2-4 in framework of CWED we have considered the theory of electron and obtained the Dirac electron equation. We have shown that the electron is the twirled *plane-polarized* semi-photon EM particle.

Below we will show that the solution of the Dirac equation in bispinor form describes also the motion of the twirled **circular-polarized** semi-photon, which is a neutral particle with half spin, but with non-zero mass. We will also show that particle and antiparticle of CWED have opposite inner spiralities. These features and any others compel us to identify this particle with neutrino.

3.0. Plane and circularly polarized electromagnetic waves

Electromagnetic waves emitted by charged particles are in general circularly (or elliptically) polarized (Ivanenko and Sokolov, 1949; Grawford, 1970). Electromagnetic waves are also transverse in the sense that associated electric and magnetic field vectors are both perpendicular to the direction of wave propagation.

Circularly polarized waves carry energy \mathcal{E} and momentum \vec{p} as well as angular momentum \vec{J} , which are defined by energy density $U = \frac{1}{8\pi}(\vec{E}^2 + \vec{H}^2)$, momentum density $\vec{g} = \frac{1}{c^2}\vec{S}_p$, and angular momentum flux density, which is given by

$$\vec{s} = \vec{r} \times \vec{g} = \frac{1}{4\pi c} r \times \vec{E} \times \vec{H}, \quad (3.1)$$

where $\vec{S}_p = \frac{c}{4\pi} \vec{E} \times \vec{H}$ is the Poynting vector indicates not only the magnitude of the energy flux density, but also the direction of energy flow. For simple electromagnetic waves, the Poynting vector is in the same direction as the wave vector. The angular momentum flux density can be checked by the circularly motion of the electron in the circularly polarized wave field (Grawford, 1970).

The figure 1 below shows propagation of electric field associated with a circularly polarized wave with positive (right) and negative (left) helicity.

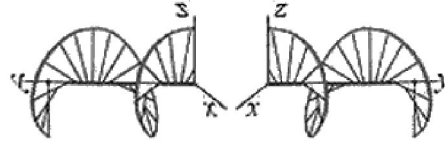


Fig. 1

Positive helicity is the case when the electric field vector is rotated such a right screw would move in the direction of wave propagation (note that in the optics it is called "left hand" circular polarization). Negative helicity (right hand polarization in optics) refers to rotation in the opposite direction. The direction of the end of the helix indicates the head of the electric field vector, which is rotating around the y axis (as on fig.1).

Since it is impossible by any transformation, except for the spatial reflection, to transfer the right (left) spiral to the left (right) spiral, the *circular polarization of photons is their integral characteristic kept at all transformations, except of the mirror transformation.*

Since the photon helicity is connected to the field rotation, in classical electrodynamics they also talk about rotation of a photon and they enter the photon rotation characteristic – the angular momentum or spin of a photon. In quantum mechanics the spin attributing of a photon has some conditional character. As it is known, as the spin is named the internal angular momentum of a particle in those systems, in which the considered particle is in rest. Therefore in case of a photon, whose speed can not be other than the light speed, it is correct to talk more about the photon helicity than about the spin (Gottfried and Weisskopf, 1984). In this case it is possible to define as helicity the vector (Grawford, 1970).

$$\vec{h}_{ph} \equiv \vec{s}_{ph} = \pm \frac{\mathcal{E}_{ph}}{\omega} \vec{p}^0, \quad (3.2)$$

where \vec{p}^0 is the unit Poynting vector, \mathcal{E}_{ph} and ω are the photon energy and circular frequency correspondingly. Apparently the angular momentum value of this vector is equal to $|\vec{h}_{ph}| = 1\hbar$.

Then according to our hypothesis the helicity vector of neutrino, as of a twirled semi-photon, should have tangential direction to the curvilinear trajectory of twirled wave motion, and its angular momentum should be equal to half of the above value $|\vec{h}_v| = \hbar/2$.

4.0. Quantum form of the circularly polarized electromagnetic wave equations

Let us consider the plane electromagnetic wave moving, for example, along y - axis. In general case such wave has two polarizations and contains the following four field vectors:
 $\vec{\Phi}(y) = \{E_x, E_z, H_x, H_z\}$

By analogy with the method, used in chapter 2, from wave equation we can obtain two equations in quantum form:

$$\Phi^+ (\hat{\alpha}_o \hat{\varepsilon} - c \hat{\alpha} \hat{p}) = 0, \quad (4.1')$$

$$(\hat{\alpha}_o \hat{\varepsilon} + c \hat{\alpha} \hat{p}) \Phi = 0, \quad (4.1'')$$

Choosing the wave function Φ as (and only as)

$$\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix}, \quad \Phi^+ = (E_x \quad E_z \quad -iH_x \quad -iH_z), \quad (4.2)$$

and putting (4.2) in (4.1) we emerge the following Maxwell equation of advanced and retarded waves :

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0 \end{array} \right\}, \quad (4.3)$$

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = 0 \end{array} \right\}, \quad (4.4)$$

The electromagnetic wave equation (and the equations (4.1) and (4.2) also) has the following harmonic solution view (in trigonometric and exponential form correspondingly):

$$\Phi_\mu = A_\mu \sin(\omega t - \vec{k}\vec{r} + \delta), \quad (4.5)$$

$$\Phi_\mu = A_\mu e^{\frac{i}{\hbar}(\varepsilon t - \vec{p}\vec{r} + \delta)}, \quad (4.6)$$

where $\mu = 1,2,3,4$, A_μ are the amplitudes and δ is the constant phase.

Putting here $A_\mu = A_0$, $\delta = 0$, we obtain the following trigonometric form of the equation solutions:

$$\left\{ \begin{array}{l} E_x = A_0 \cos(\omega t - ky) \\ H_z = -A_0 \cos(\omega t - ky) \\ E_z = -A_0 \sin(\omega t - ky) \\ H_x = -A_0 \sin(\omega t - ky) \end{array} \right\}, \quad (4.7')$$

$$\left\{ \begin{array}{l} E_x = A_0 \cos(\omega t - ky) \\ H_z = A_0 \cos(\omega t - ky) \\ E_z = -A_0 \sin(\omega t - ky) \\ H_x = A_0 \sin(\omega t - ky) \end{array} \right\}, \quad (4.7'')$$

Let us show that the vectors \vec{E} and \vec{H} rotate in the XOZ plane. Actually, putting $y = 0$ we obtain:

$$\vec{E} = E_x \vec{i} + E_z \vec{k} = A_0 (\vec{i} \cos \omega t - \vec{k} \sin \omega t), \quad (4.8)$$

$$\vec{H} = H_x \vec{i} + H_z \vec{k} = A_0 (-\vec{i} \sin \omega t - \vec{k} \cos \omega t), \quad (4.8'')$$

and

$$\vec{E} = E_x \vec{i} + E_z \vec{k} = A_0 (\vec{i} \cos \omega t - \vec{k} \sin \omega t), \quad (4.9')$$

$$\vec{H} = H_x \vec{i} + H_z \vec{k} = A_0 (\vec{i} \sin \omega t + \vec{k} \cos \omega t) \quad (4.9'')$$

where \vec{i} , \vec{k} are the unit vectors of the OX and OZ axes. It is not difficult to show by known algebraic analysis (Jackson, 1999) that we have obtained the cyclic polarised wave. But to keep in evidence we will analyse these relations from geometrical point of view.

Taking into account that the Poynting vector defines the direction of the wave motion:

$$\vec{S}_p = \frac{c}{4\pi} \vec{E} \times \vec{H} = -\vec{j} \frac{c}{4\pi} (E_x H_z - E_z H_x), \quad (4.10)$$

where \vec{j} is the unit vectors of the OY axis, and calculating the above (4.10), we have for (4.13') and (4.13'')

$$\vec{S}_p = \frac{c}{4\pi} A_0^2 \vec{j}, \quad (4.11)$$

$$\vec{S}_p = -\frac{c}{4\pi} A_0^2 \vec{j}, \quad (4.12)$$

correspondingly. Thus, the photons of the right and left systems (4.8') and (4.9) move in the contrary directions.

Fixing the vector \vec{E}, \vec{H} positions in two successive time instants ($t_0 = 0$ and $t_1 = t_0 + \Delta t$), we can define the rotation direction. The results are imaged on the figures 2 and 3 correspondingly:

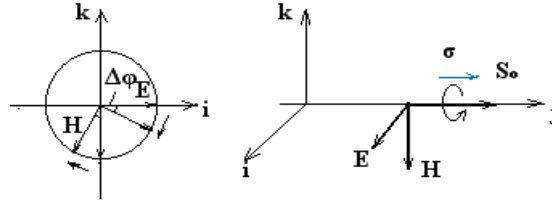


Fig. 2

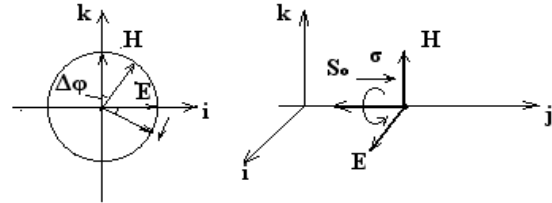


Fig. 3

As we can see the equation sets (4.3 and (4.4) describe the waves with right and left circular polarization correspondingly.

Obviously, by the ring twirling of the circular polarized photon, its helicity does not disappear, but inside the torus become poloidal helicity (or "p-helicity"). At the same time, the movement of fields of a photon along a circular trajectory forms other characteristics of an elementary particle - namely the angular momentum of a particle, or spin. Apparently, the spin of a massive particle and its poloidal angular momentum (p-helicity) are different characteristics. Since these characteristics are the own internal characteristics of a photon, *in the non-linear electromagnetic theory the spin and the poloidal helicity of a particle are independently conserved values.*

We will show now that the Dirac equation with mass term can be considered as the equation of the twirled circularly polarized waves.

5.0. The equation of massive neutrino of CWED

Apparently the massive neutrino must be described by the Dirac lepton equation with the mass term. Consider the mass term appearing in this case.

Let the circular-polarized wave \vec{E}, \vec{H} , which have the field components $\{E_x, E_z, H_x, H_z\}$, be twirled with some radius r_k in the plane (X', O', Y') of a fixed co-ordinate system

(X', Y', Z', O') so that E_x, H_x are parallel to the plane (X', O', Y') and E_z, H_z are perpendicular to it (see fig. 4)

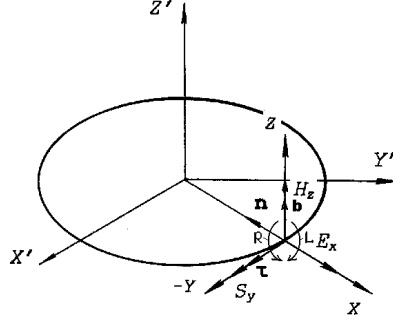


Fig. 4

where the circular arrows shows the right (R) and the left (L) rotation of photon fields.

Let's replace here the unit vectors of the Cartesian coordinate system axes of paragraph 3 $\{\vec{1}_x, \vec{1}_y, \vec{1}_z\}$, connected with a wave, by the vectors of the Frenet-Serret trihedron $\{\vec{n}, \vec{\tau}, \vec{b}\}$ accordingly. Then for the electric and magnetic vectors we have instead of (3.4):

$$\vec{E}(y, t) = \vec{n}E_x + \vec{b}E_z = (\vec{n}E_{x0} + \vec{b}E_{z0})e^{i\omega t}, \quad (5.1)$$

$$\vec{H}(y, t) = \vec{n}H_x + \vec{b}H_z = (\vec{n}H_{x0} + \vec{b}H_{z0})e^{i\omega t}, \quad (5.2)$$

Here, as well as in the case of the linear polarized EM strings, the unit vector of a normal \vec{n} turns around of $O'Z'$ axis, and bivector \vec{b} remains parallel to it

By analogy to the procedure stated in chapter 2 it is easy to receive the equations of the twirled semi-photons, i.e. the Dirac equation with a mass term.

Unlike to a case of twirling of the plane polarized photons, considered in chapter 2, we do not have basis in advance to approve that here magnetic currents are equal to zero. Really, in first case the magnetic vector was parallel to the rotation axis and kept a constant direction in space, and electric continuously turned around of it, changing direction in space. In the given case the magnetic vector rotates around of a trajectory of motion and is transported along a trajectory just as an electric vector.

Using the procedure stated in chapter 2, we shall show, that in this case there are both electric and magnetic currents.

Let's in the equations of the initial EM strings (4.4) consider the expressions $\vec{j}^e = \frac{1}{4\pi} \frac{\partial \vec{E}}{\partial t}$

and $\vec{j}^m = \frac{1}{4\pi} \frac{\partial \vec{H}}{\partial t}$. (Remind that after twirling of EM string, a field vectors of an initial

wave \vec{E}, \vec{H} are transformed in a field vectors of the twirled wave, designated by us in the electromagnetic form as \vec{E}, \vec{H} , and in the quantum form as ψ). Taking into account that $\frac{\partial \vec{b}}{\partial t} = 0$,

we shall receive from (5.1) and (5.2):

$$\frac{\partial \vec{E}}{\partial t} = -\frac{\partial E_x}{\partial t} \vec{n} + \frac{\partial E_z}{\partial t} \vec{b} - E_x \frac{\partial \vec{n}}{\partial t}, \quad (5.3)$$

$$\frac{\partial \vec{H}}{\partial t} = \frac{\partial H_x}{\partial t} \vec{n} + \frac{\partial H_z}{\partial t} \vec{b} + H_x \frac{\partial \vec{n}}{\partial t}, \quad (5.4)$$

where $\frac{\partial \vec{n}}{\partial t} = -c\kappa\vec{\tau} = -\frac{c}{r_C}\vec{\tau}$, and $r_C = \hbar/mc$. Thus, we receive the electric and magnetic

tangential currents, a particularity of which is that they are **alternating**:

$$\vec{j}_\tau^e = \frac{\omega_p}{4\pi} E_x \cdot \vec{\tau} = \frac{\omega_p}{4\pi} E_{x0} \cdot \vec{\tau} \cdot \cos \omega t, \quad (5.5)$$

$$\vec{j}_\tau^m = -\frac{\omega_K}{4\pi} H_x \cdot \vec{\tau} = -\frac{\omega_K}{4\pi} H_{x0} \cdot \vec{\tau} \cdot \cos \omega t, \quad (5.6)$$

Now it is not difficult to see that the quantum form of the equation of circular polarized semi-photons with opposite spirality are the Dirac equations with mass terms.

Taking into account the previous section results, we obtain the twirled semi-photon equations in quantum form, which are equivalent to the Dirac equations:

$$\frac{\partial \psi}{\partial t} - c \hat{\alpha} \vec{\nabla} \psi - i \hat{\beta} \frac{c}{r_C} \psi = 0, \quad (5.7')$$

$$\frac{\partial \psi}{\partial t} + c \hat{\alpha} \vec{\nabla} \psi + i \hat{\beta} \frac{c}{r_C} \psi = 0, \quad (5.7'')$$

and electromagnetic form of these equations:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = ij_x^m \end{array} \right. , (5.8') \quad \left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_z}{\partial y} = -ij_x^e \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_x}{\partial y} = 0 \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_z}{\partial y} = ij_x^m \end{array} \right. , (5.8'')$$

which are the complex Maxwell equations with *imaginary tangential alternating currents - electric and magnetic*.

We can schematically represent the fields' motion of particles, described by these equations, in the following way (fig. 5):

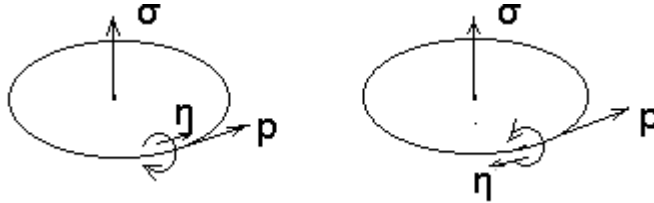


Fig. 5

According to figs. 2 and 3 the semi-photons (fig. 5) have the contrary p-helicities. In the first case the helicity vector and the Poynting vector have the same directions; in the second case they are contrary. Therefore in the non-linear theory we can define the *inner or p-helicity* as the projection of the poloidal rotation momentum on the momentum of the ring field motion.

It is not difficult to show (Davydov, 1965) that actually the helicity is described in CWED by matrix $\hat{\alpha}_5$. Multiplying the Dirac equation on $i\hat{\alpha}_5\hat{\beta}$ and taking in account that $i\hat{\alpha}_5\hat{\beta}\vec{\alpha} = \hat{\sigma}$,

where $\hat{\sigma}' = \begin{pmatrix} \hat{\sigma} & 0 \\ 0 & \hat{\sigma} \end{pmatrix}$ are the spin matrix, and $\hat{\beta}\hat{\alpha}_5 = -\hat{\alpha}_5\hat{\beta}$, $\hat{\beta}^2 = 1$, we obtain:

$$(i\hat{\beta}\hat{\alpha}_5\hat{e} + c\hat{\sigma}'\hat{p} - imc^2\hat{\alpha}_5) \psi = 0, \quad (5.9)$$

$$(i\hat{\beta}\hat{\alpha}_5\hat{e} - c\hat{\sigma}'\hat{p} + imc^2\hat{\alpha}_5) \psi = 0, \quad (5.9'')$$

from which we can emerge for the helicity matrix the following expressions:

$$\hat{\alpha}_5 = \frac{c \hat{\sigma}' \vec{p}}{i(\hat{\beta} \hat{\epsilon} + mc^2)}, \quad (5.10')$$

$$\hat{\alpha}_5 = \frac{-c \hat{\sigma}' \vec{p}}{i(\hat{\beta} \hat{\epsilon} - mc^2)}, \quad (5.10'')$$

that connects the $\hat{\alpha}_5$ matrix with helicity, but it is not as in the case $m = 0$.

According to our theory inside the particle the operator $\hat{\alpha}_5$ describes the poloidal rotation of the fields (fig. 5). Remembering that according to the electromagnetic interpretation (see the chapter 3) the value $\psi^+ \hat{\alpha}_5 \psi$ is the pseudoscalar of electromagnetic theory $\psi^+ \hat{\alpha}_5 \psi = \vec{E} \cdot \vec{H}$, we can affirm, that in the CWED the p-helicity is the Lorentz-invariant value for the massive particles, and actually it is the origin of the parity non-conservation of the massive particles.

A question arises about how the twirled semi-photon can has a mass and simultaneously doesn't have a charge. It is easy to understand the origin of this difference.

6.0. The charge and mass of EM neutrino

6.1. Demonstration of the neutrino charge absence

The charge is defined by integral on some volume from a current density, which is proportional to the first power of field strength. Obviously, it is possible to have a case when the subintegral expression is not equal to zero, but the integral itself is equal to zero. It is easy to check that we will receive such a result in case when subintegral function changes according to the harmonious law.

It is not difficult to calculate the charge density of the twirled semi-photon particle:

$$\rho_p = \frac{j_\tau}{c} = \frac{1}{4\pi} \frac{\omega_p}{c} E = \frac{1}{4\pi} \frac{1}{r_p} E, \quad (6.1)$$

The full charge of the twirled semi-photon can be defined by integrating

$$q = \int_{\Delta\tau_i} \rho_p d\tau, \quad (6.2)$$

where $\Delta\tau_i$ is the volume of particle fields.

Using the fig. 4 and taking $\vec{E} = \vec{E}(l)$, where l is the length along the circular trajectory, we obtain:

$$q = \frac{1}{4\pi} \frac{\omega_p}{c} E_o \int \cos k_p l d\tau = 0, \quad (6.3)$$

(here E_o is the amplitude of the twirled photon wave field, S_c - the area of torus cross-section, ds is the element of the cross-section surface, dl - the element of the length, $k_p = \frac{\omega_p}{c}$ - the wave-vector).

It is easy to understand these results: because the ring current is alternate, the full charge is equal to zero.

6.2. Demonstration of the neutrino mass presence

The particle mass is defined by integral from energy density, which is proportional to the second power of the field strength. In this case the integral is always distinct from zero if the field is distinct from zero.

To calculate the mass we must calculate first the energy density of the electromagnetic field:

$$\rho_\epsilon = \frac{1}{8\pi} (\vec{E}^2 + \vec{H}^2), \quad (6.4)$$

In linear approximation (in Gauss's system) we have $|\vec{E}| = |\vec{H}|$. Then (6.4) can be written so:

$$\rho_\epsilon = \frac{1}{4\pi} E^2, \quad (6.5)$$

Using (6.5) and a well-known relativistic relationship between a mass and energy densities:

$$\rho_m = \frac{1}{c^2} \rho_\varepsilon, \quad (6.6)$$

we obtain:

$$\rho_m = \frac{1}{4\pi c^2} E^2 = \frac{1}{4\pi c^2} E_o \cos^2 k_s l, \quad (6.7)$$

Using (6.7), we can write for the semi-photon mass:

$$m_s = \int \rho_m d\tau = \frac{E_o^2}{\pi c^2} \int \cos^2 k_s l d\tau \neq 0, \quad (6.8)$$

Obviously this expression never can be equal to zero. Thus actually in the framework of CWED there are the cases when the particle mass do not equal to zero, but the particle charge is equal to zero.

7.0. Topological peculiarities of neutrino-like particle structure

According to our analysis the lepton are the twirled half-periods of photon. In this case neutrino as twirled helicoid represents the Moebius's strip: its field vectors (electric and magnetic) at the end of one coil passes to a state with opposite direction in comparison with the twirled photon vectors, and only by two coils, the vector comes to the starting position (see fig. 6)

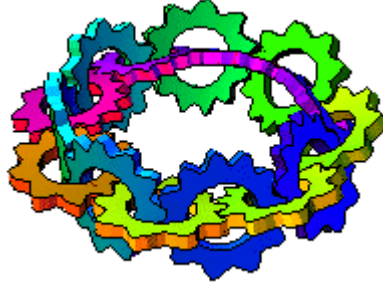


Fig. 6

(fig. 6 is from from (Moebius Strip, MathWorld): <http://mathworld.wolfram.com/MoebiusStrip.html> , where the animation shows a series of gears arranged along a Möbius strip as the electric and magnetic field vectors motion)

Strict verification of the above conclusion about neutrino fields structure follows from the analysis of transformation properties of the twirled semi-photon wave function. (Remember also that a plane electromagnetic wave can be considered as vector combination of two circularly polarized waves rotating in opposite directions).

As it is shown in quantum field theory (Gottfried and Weisskopf, 1984; Ryder, 1985), the rotation matrix possesses a remarkable property (see also section 6 of the chapter 2), which is illustrated particularly visual by analysis of the neutrino structure.

If the rotation occurs on the angle $\theta = 2\pi$ around any axis (therefore occurs the returning to the initial system of reference) we find, that $U = -1$, instead of $U = 1$ as it was possible to expect. Differently, the state vector of system with spin half, in usual three-dimensional space has ambiguity and passes to itself only after turn to the angle 4π (which accords here to the one wave length of electromagnetic wave).

From above it follows that semi-photon can appear only in CWED, and in classical linear electrodynamics it do not exist.

8.0. Pauli exclusion principle

As we already mention, according to R. Feynman particle, which has the Moebius strip topology, must obey the Pauli exclusion principle. Let show this.

The Pauli exclusion principle can be written in following form: *particles of half-integer spin have antisymmetric wavefunctions, and particles of integer spin have symmetric wavefunctions.*

The answer to the question (Feynman, Leighton, and Sands, 1963), why "particles with half-integral spin are Fermi particles whose amplitudes add with the minus sign", underlies the Fermi statistics, and therefore the Pauli exclusion principle.

There is (Gottfried and Weisskopf, 1986; Gould, 1995) a remarkable property of lepton in three dimensional space: when a lepton is rotated 360 degrees (what means that the wave function phase shifts on 360 degrees), it returns to a state that looks the same geometrically, but that is topologically distinct with respect to its surroundings: a twist has been introduced. A second full rotation (a total of 720 degrees) brings the object back to its original state.

In his last lecture R. Feynman (Feynman, 1987) sketched an elementary argument for above question (see fig 7, which was taken from R. Feynman paper):

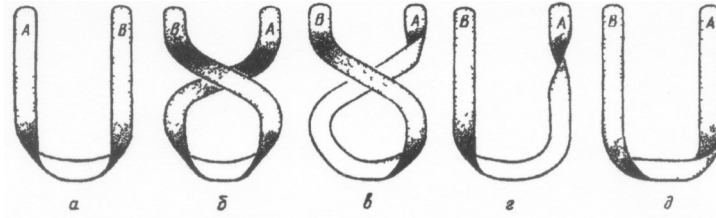


Fig. 7

To see this, first grasp the two ends of a belt, one end in each hand; then interchange the position of your hands. So we have introduced a "twist", which is topologically equivalent to having rotated one end of the belt by 360 degrees.

Thus, when fermions are interchanged, one must keep track of this "implied rotation" and the phase shift, sign change, and destruction interference to which it gives rise. For example, if $A(1)B(2)$ describes "electron 1 in state A and electron 2 in state B," then the state with electrons interchanged must be $-A(2)B(1)$ and their superposition is $A(1)B(2) - A(2)B(1)$

Since in the framework of CWED the leptons have the topology of Moebius strip, they must behave as fermions of quantum field theory.

Note also that in CWED two neutrino with left and right poloidal helicity form one twirled circular polarized photon. This corresponds to the theory of Luis de Broglie about the neutrino nature of light (Broglie, 1932a; 1932b; 1934) if we mean the twirled photon, not the "linear".

9.0. The neutrino of SM and the neutrino of CWED

Let's compare the features of neutrino of CWED with neutrino of SM.

1. it is a lepton, i.e. is described by the Dirac equation;
2. it doesn't have electric charge;
3. particles and antiparticles are distinguished only by helicity;
4. it has all necessary invariant properties according to the theory of the weak interaction.

On the other hand, the CWED neutrino has mass, and the SM neutrino is strictly massless and as such it is described by the Weyl equation. But modern experiments indicate that this statements is doubtful. Thus, a change of SM is necessary, which doesn't violate other advantage of SM.

It was shown that the internal motion of semi-photon fields of neutrino on a circular trajectory is described by the Dirac lepton equation with mass equal to zero, i.e. by the Weil equation. In other words, in framework of CWED the Weyl equation is the equation of internal motion of a neutrino fields as the massless particle (photon fields). As it follows from solution of the Weyl equation and as we have shown above, the internal poloidal rotation (p-helicity) allows the massive neutrino to have properties of the massless neutrino. On the other hand from the outside a "stopped" twirled electromagnetic wave look as the massive neutrino.

If we identify the CWED neutrino with the neutrino of the Standard Model, we eliminate the difficulties of the theory of Standard Model with minimum alteration of the theory. Actually, all that is necessary to overcome the difficulties is to recognize that the neutrino has an internal motion, described by the Weyl equation.

Chapter 6. On hadrons theory

As it is known the hadron theory is based on the Yang-Mills equation.

1.0. Introduction. Dirac and Yang-Mills equations of SM

As it follows from the Standard Model theory (Pich, 2000; Peak and Varvell ; Okun, 1982). the quark family is analogue to the lepton family and the Yang-Mills equation is the generalisation of the Dirac electron equation.

The Dirac equation for the electron in the external field can be written in the form (Schiff, 1955):

$$\hat{\alpha}_\mu (\hat{p}_\mu + p_\mu^e) \psi + \hat{\beta} m_e c^2 \psi = 0 \quad (1.1)$$

where $\mu = 0,1,2,3$, $\hat{p}_\mu = \{\hat{\epsilon}, c\hat{p}\}$, where $\hat{\epsilon} = i\hbar \frac{\partial}{\partial t}$, $\hat{p} = -i\hbar \vec{\nabla}$ are the operators of energy

and momentum, respectively; $\hat{p}_\mu^e = \{\hat{\epsilon}_{ex}^e, c\hat{p}_{ex}^e\} = j_\mu A_\mu$, where $\epsilon_{ex}^e = e\phi$, $\vec{p}_{ex}^e = \frac{e}{c} \vec{A}$ are the electron energy and momentum in the external electromagnetic field; respectively; (ϕ, \vec{A}) is 4-potential of the external field; c is the light velocity, $-e$, m are the electrical charge and mass of the electron correspondingly.

In Quantum Chromodynamics, which is described by Yang-Mills equation, we have quarks instead of electrons, and gluons instead of photons, between which there are the strong interactions instead of the electromagnetic interactions. The Yang-Mills equation for one quark may be written (Pich, 2000; Peak and Varvell ; Okun, 1982) similarly to (1.1):

$$\hat{\alpha}_\mu (\hat{p}_\mu + p_\mu^q) \psi_q + \hat{\beta} m_q c^2 \psi_q = 0, \quad (1.2)$$

where ψ_q are the quark fields, $p_\mu^q \equiv icg\vec{G}_\mu$ with $\vec{G}_\mu = \frac{1}{2} \sum_{a=1}^8 G_\mu^a \lambda_a$ is the potential of the gluon field, λ_a, g, m_q are the Gell-Mann matrices, strong charge and quark mass, respectively.

2.0. “One quark” theory of hadrons

Formally we can say (Peak and Varvell ; Okun, 1982), that hadron is described by two or three Dirac electron equations of (1.1) type. Thus, conditionally we can name the Dirac electron equation as the “one quark” equation.

But here we need to take into account that the Dirac equation (1.1) is not the free electron equation. On the other hand, the equation (1.2) is indeed the equation of the “free” quark. The external field terms are used in the QED for the description of the interaction between the electron and other particles. The similar terms in the Yang-Mills equation are the internal field, which describes the quark-quark interaction of the same hadron.

3.0. The derivation of Yang-Mills equation in framework of CWED

In the present chapter we will show that the CWED representation allows us to interpret the Yang-Mills equation as the *curvilinear electromagnetic waves superposition*.

Obviously, to obtain the Yang-Mills equation we must sum three “one quark” equations without mass and “turn on” the twirling of the fields.

3.1. Electromagnetic forms of “three quark” equations

As the Pauli matrices are (Ryder, 1985) the generators of the 2D rotation, for the “three quark” electromagnetic representation we must use the generators of the 3D rotation, which are the known photon spin 3x3-matrices \hat{S} of the O(3) group [3,12]:

$$\hat{S}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \hat{S}_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \hat{S}_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.1)$$

As the “three quark” equations for the particle and antiparticle we will use the Dirac equations (1.1) in the following form:

$$\begin{aligned} & \left[\left({}^6\hat{\alpha}_0 \hat{\epsilon} - c {}^6\hat{\alpha} \hat{p} \right) - {}^6\hat{\beta} mc^2 \right] \psi = 0 \\ & \psi^+ \left[\left({}^6\hat{\alpha}_0 \hat{\epsilon} + c {}^6\hat{\alpha} \hat{p} \right) + {}^6\hat{\beta} mc^2 \right] = 0 \end{aligned} \quad (3.2)$$

where the left upper index “6” means that these matrices are the 6x6-matrices of the following type:

$${}^6\hat{\alpha} = \begin{pmatrix} \hat{0} & \hat{S} \\ \hat{S} & \hat{0} \end{pmatrix}, {}^6\hat{\alpha}_0 = \begin{pmatrix} \hat{S}_0 & \hat{0} \\ \hat{0} & \hat{S}_0 \end{pmatrix}, {}^6\hat{\alpha}_4 \equiv {}^6\hat{\beta} = \begin{pmatrix} \hat{S}_0 & \hat{0} \\ \hat{0} & -\hat{S}_0 \end{pmatrix}, \quad (3.3)$$

Here $\hat{S}_0 = \hat{1}$ and wave function ${}^6\psi = \begin{pmatrix} \vec{E} \\ i\vec{H} \end{pmatrix}$ is the 6x1 matrix.

As it is not difficult to test the above matrices give the right electromagnetic expressions of the bilinear form of the theory:

$$\text{the energy: } {}^6\psi^+ {}^6\hat{\alpha}_0 {}^6\psi = \vec{E}^2 + \vec{H}^2 = 8\pi U,$$

$$\text{the Poynting vector (or momentum): } \vec{S}_p = \frac{1}{8\pi} {}^6\psi^+ {}^6\vec{\alpha} {}^6\psi,$$

$$\text{and the 1st scalar of the EM field: } {}^6\psi^+ {}^6\hat{\alpha}_4 {}^6\psi = 2(\vec{E}^2 - \vec{H}^2) = 4\pi F_{\mu\nu}F^{\mu\nu}.$$

3.2. “Three-quarks” equation without mass-interaction terms

From the above follows that the proton equation can be represented by three “one quark” equations, i.e. three electron equations or three pairs of the scalar Maxwell equations (one pair for each coordinate). Obviously, there is a possibility of two directions of rotations of each quark (the left and the right quarks). Therefore, the 6+6 scalar equations for proton description must exist as well as the 6+6 equations for the antiproton description.

Let us find at first these equations without mass-interaction, putting the mass-interaction terms equal to zero. Using (3.3) from the equations (3.2) we obtain the Maxwell equation without current:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = 0, \quad a \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = 0, \quad a' \\ \frac{1}{c} \frac{\partial E_y}{\partial t} - \frac{\partial H_x}{\partial z} = 0, \quad b \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_y}{\partial z} = 0, \quad b' \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} = 0, \quad c \\ \frac{1}{c} \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = 0, \quad c' \end{array} \right. \quad (3.4)$$

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = 0, \quad a \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = 0, \quad a' \\ \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_y}{\partial z} = 0, \quad b \\ \frac{1}{c} \frac{\partial H_y}{\partial t} + \frac{\partial E_x}{\partial z} = 0, \quad b' \\ \frac{1}{c} \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} = 0, \quad c \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} = 0, \quad c' \end{array} \right. \quad (3.5)$$

As it is not difficult to see that each pair of the equations a, b, c describes a separate ring; the fields vectors of equations (3.4) are rolled up in the plains XOZ , ZOY , YOX , and similarly the fields vectors of the equations (3.5) are rolled in the plains XOY , YOZ , ZOX .

3.3. The compensation or gauge fields in the modern theory

The modern particle theory is also known as the gauge field theory because the interactions between the particles are introduced in the field equation via the gauge transformations. It is known (Kaempffer, 1965; Ryder, 1985) that this procedure is mathematically equivalent to the field vector transformations in the curvilinear space, which lead to the covariant derivative appearance.

It is not difficult to show (Ryder, 1985), that the electromagnetic field appears naturally as a consequence of the requirement of the Lagrangian invariance relatively to the gauge transformations of the local rotations in the internal space of the complex field ψ , when the Lagrangian has the symmetry $O(2)$ or $U(1)$. Mathematically this is expressed through the replacement of the simple derivatives with the covariant derivatives.

The generalization of this result on a case of 3D-space is the Yang - Mills field. The elementary generalization of this symmetry is the non-abelian group $SU(2)$; i.e. the question is about the theory of the non-abelian gauge fields.

Let's consider (see details in (Ryder, 1985)) the rotation of some field vector \vec{F} in three-dimensional space around some axis on an infinitesimal angle. Here the value $|\vec{\varphi}|$ is a rotation angle, and the vector $\vec{\varphi}/|\vec{\varphi}|$ sets the direction of the axis of rotation. The transition from the initial position of a vector to the final position will be defined by the transformation:

$$\vec{F} \rightarrow \vec{F}' = \vec{F} - \vec{\varphi} \times \vec{F}, \quad (3.6)$$

The problem is to create independent rotations in various points of space. In order to construct correctly the covariant field derivative, we should make parallel transport of the vectors into the space, instead of on a flat curve, as in the above case of spinorial theory. The corresponding analysis (Ryder, 1985) allows us to receive an expression similar to the expression, which appears by the spinor transport on a flat curve.

It can be shown also (Kaempffer, 1965; Ryder, 1985) that this expression defines a covariant derivative of the field ψ , which is transformed according to some representation of a group:

$$\frac{D\psi}{dx^\mu} = D_\mu \psi = \left(\partial_\mu - igM^a A_\mu^a \right) \psi, \quad (3.7)$$

where the matrixes M^a are the generators of the rotation. It is not difficult to make sure that this expression gives the same covariant derivative, as found earlier in the case of electron theory, and can give the mass-interaction terms.

In the following section we will consider the electromagnetic discription of the mass-interaction term appearance.

3.3.1. The electromagnetic description of the mass-interaction term appearance

The spinorial theory shows that the appearance of the internal mass-interaction terms is bounded with the three vectors $\vec{E}, \vec{H}, \vec{S}_p$, moving along the curvilinear trajectory. These vectors represent the moving trihedral of the Frenet-Serret (Gray, 1997). In the general case, when the electromagnetic wave field vectors of three-quark particles move along the space curvilinear trajectories, not only the additional term, defined by the curvature, appears, but also the terms that are defined by the torsion of the trajectory.

Actually, in this case we have:

$$\begin{aligned} \frac{\partial \vec{E}}{\partial t} &= -\frac{\partial E}{\partial t} \vec{n} - E \frac{\partial \vec{n}}{\partial t} \\ \frac{\partial \vec{H}}{\partial t} &= \frac{\partial \vec{H}}{\partial t} \vec{b} + H \frac{\partial \vec{b}}{\partial t} \end{aligned} \quad (3.8)$$

where \vec{b} is the binormal vector. According to the Frenet-Serret formulas we have:

$$\begin{aligned} \frac{\partial \vec{n}}{\partial t} &= -v_p \mathbf{K} \vec{\tau} + v_p \mathbf{T} \vec{b} \\ \frac{\partial \vec{b}}{\partial t} &= -v_p \mathbf{T} \vec{n} \end{aligned} \quad (3.9)$$

where $\mathbf{T} = \frac{1}{r_T}$ is the torsion of the trajectory and r_T is the torsion radius. Thus, the displacement currents can be written in the form:

$$\begin{aligned} \vec{j}^e &= -\frac{1}{4\pi} \frac{\partial E}{\partial t} \vec{n} + \frac{1}{4\pi} \omega_K E \cdot \vec{\tau} - \frac{1}{4\pi} \omega_T E \cdot \vec{b} \\ \vec{j}^m &= \frac{1}{4\pi} \frac{\partial H}{\partial t} \vec{b} - \frac{1}{4\pi} \omega_T H \cdot \vec{n} \end{aligned} \quad (3.10)$$

where $\omega_T = \frac{v_p}{r_T} \equiv c\mathbf{T}$ we name the torsion angular velocity.

Thus, we can obtain the following electromagnetic representation of the three-quarks' equations:

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_x}{\partial t} - \frac{\partial H_z}{\partial y} = j_1^e, \quad a \\ \frac{1}{c} \frac{\partial H_z}{\partial t} - \frac{\partial E_x}{\partial y} = j_1^m, \quad a' \\ \frac{1}{c} \frac{\partial E_y}{\partial t} - \frac{\partial H_x}{\partial z} = j_2^e, \quad b \\ \frac{1}{c} \frac{\partial H_x}{\partial t} - \frac{\partial E_y}{\partial z} = j_2^m, \quad b' \\ \frac{1}{c} \frac{\partial E_z}{\partial t} - \frac{\partial H_y}{\partial x} = j_3^e, \quad c \\ \frac{1}{c} \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} = j_3^m, \quad c' \end{array} \right. \quad (3.11)$$

$$\left\{ \begin{array}{l} \frac{1}{c} \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} = j_1^e, \quad a \\ \frac{1}{c} \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} = j_1^m, \quad a' \\ \frac{1}{c} \frac{\partial E_x}{\partial t} + \frac{\partial H_y}{\partial z} = j_2^e, \quad b \\ \frac{1}{c} \frac{\partial H_y}{\partial t} + \frac{\partial E_x}{\partial z} = j_2^m, \quad b' \\ \frac{1}{c} \frac{\partial E_y}{\partial t} + \frac{\partial H_z}{\partial x} = j_3^e, \quad c \\ \frac{1}{c} \frac{\partial H_z}{\partial t} + \frac{\partial E_y}{\partial x} = j_3^m, \quad c' \end{array} \right. \quad (3.12)$$

where j_k ($k = 1, 2, 3$) are the currents of each quark:

$$\vec{j}_k^e = \frac{\omega_{Kk}}{4\pi} E\vec{\tau} + \frac{\omega_{Tk}}{4\pi} E\vec{b} \quad \text{and} \quad \vec{j}_k^m = \frac{\omega_{Tk}}{4\pi} H\vec{n}, \quad (3.13)$$

As we noted at the analysis of electromagnetic representation of the electron equation, the charge, mass and interaction between particles arise simultaneously during twirling and division of a photon. In other words, the appearance of currents at twirling a linear photon simultaneously describes the appearance of a charge, masses and electron interactions.

Since in this case we have, conditionally speaking, three electromagnetic electron equations, it is necessary to conclude, that EM masses of quarks, their charges and interactions between them are described by nine currents of the equations (3.11) or (3.12). It is possible to assume, that three from them which are tangential electric currents, define charges of quarks and partially the masses. Whether the others currents (three electric binormal and three magnetic normal) insert some amendments into these parameters, it is difficult to tell.

3.3.2. The description of of the mass-interaction term appearance in the framework of Riemann geometry

The appearance of additional term follows from the general theory of the vector motion along the curvilinear trajectory. This theme was studied in the vector analyse, in the differential geometry and in the hypercomplex number theory hundred years ago (Madelung, 1957; Korn and Korn, 1961) and it is well known. Below we consider some conclusions of these theories.

Any vector $\vec{F}(\vec{r}, t)$ can have the following forms (Korn and Korn, 1961):

$$\begin{aligned} \vec{F}(\vec{r}, t) &= \vec{F}(x^0, x^1, x^2, x^3) = F^0 \vec{e}_0 + F^1 \vec{e}_1 + F^2 \vec{e}_2 + F^3 \vec{e}_3 = \\ &= F_0 \vec{e}^0 + F_1 \vec{e}^1 + F_2 \vec{e}^2 + F_3 \vec{e}^3 \end{aligned} \quad (3.17)$$

where $F^0, F^1, F^2, F^3, F_0, F_1, F_2, F_3$ are the invariant and co-variant vector modulus and \vec{e}^i and \vec{e}_i are the basis vectors, which in general case are changed from point to point. When vector moves along the curvilinear trajectory the partial derivatives get the view:

$$\frac{\partial \vec{F}}{\partial x^j} = \frac{\partial F^i}{\partial x^j} \vec{e}_i + F^i \frac{\partial \vec{e}_i}{\partial x^j} = \frac{\partial F_i}{\partial x^j} \vec{e}^i + F_i \frac{\partial \vec{e}^i}{\partial x^j}, \quad (3.14)$$

where the following notations are used:

$$\frac{\partial \vec{e}_i}{\partial x^j} = \Gamma_{ij}^k \vec{e}_k = -\Gamma_{kj}^i \vec{e}^k, \quad (3.15)$$

(here $i, j, k, = 0, 1, 2, 3$)

The coefficients Γ_{ij}^k are named Christoffel symbols or bound coefficients. Thus, for the y -direction photon

$$\begin{cases} \vec{E} = E_3 \vec{e}^3 \\ \vec{H} = H_1 \vec{e}^1 \end{cases}, \quad (3.16)$$

we obtain:

$$\begin{cases} \frac{1}{c} \frac{\partial \vec{E}}{\partial t} = \frac{\partial E_3}{\partial x^0} \vec{e}^3 + E_3 \Gamma_{k0}^3 \vec{e}^k \\ \frac{1}{c} \frac{\partial \vec{H}}{\partial t} = \frac{\partial H_1}{\partial x^0} \vec{e}^1 + H_1 \Gamma_{k0}^1 \vec{e}^k \\ \frac{\partial \vec{E}}{\partial y} = \frac{\partial E_3}{\partial x^2} \vec{e}^3 + E_3 \Gamma_{k2}^3 \vec{e}^k \\ \frac{\partial \vec{H}}{\partial y} = \frac{\partial H_1}{\partial x^2} \vec{e}^1 + H_1 \Gamma_{k2}^1 \vec{e}^k \end{cases}, \quad (3.17)$$

The same we can obtain for the other directions of the photons.

Thus, in the general case, when the electromagnetic field vectors of three-knot particles move along the curvilinear trajectories, the additional terms of the same type, which we obtained in the case of Yang-Mills equation, appear.

Note: in the framework of CWED the Christoffel symbols are not the abstract mathematical values. On one hand they are the physical values; namely, they are the currents, which appeared thanks to the twirling and torsion of the electromagnetic vectors. On the other hand they have geometrical sense: they are proportional to the curvature of the trajectory \mathbf{K} and to the torsion of the trajectory \mathbf{T} .

4.0. The introduction of the terms of interactions of quarks

Let us examine the formation of hadrons (for example, proton) from the point of view of the reaction of photoproduction

$$\gamma + N \rightarrow p^+ + p^- + N, \quad (4.1)$$

where γ, p^+, p^- are a gamma-quantum (photon), proton and antiproton respectively, and N is the nuclear field, in which is accomplished the symmetry breaking of photon and as consequence the appearance of massive particles. It should conclude from (4.1) that quarks themselves are produced simultaneously with interaction between them. Remember that we had the same in the case of the photoproduction of electron-positron pair with the only difference that in the last case the interaction was external. As the consequence of this we obtained the doubled value of mass member. Consequently, instead of (3.2) we will have

$$\begin{aligned} & \left[\left({}^6\hat{\alpha}_o \hat{\varepsilon} - c \cdot {}^6\hat{\alpha} \hat{p} \right) - 2 \cdot {}^6\hat{\beta} m_l c^2 \right] \psi = 0 \\ & \psi^+ \left[\left({}^6\hat{\alpha}_o \hat{\varepsilon} + c \cdot {}^6\hat{\alpha} \hat{p} \right) + 2 \cdot {}^6\hat{\beta} m_l c^2 \right] = 0 \end{aligned}, \quad (4.2)$$

where here through m_l ($l = 1, 2, \dots, 9$) we conditionally designate the appropriate mass and currents, which describe both the quarks and the gluons. According to (4.2) we have 9 quark currents and 9 gluon currents, so that the summary energies each of these currents $W = \sum_{l=1}^{l=9} m_l c^2$ must be

the same, or in other words summary energy of proton is divided in half between the quarks and the gluons. It is not also difficult to explain, why the inner virtual photons, called gluons, inside the hadron acquire the currents: in the strong intrinsic field of quarks they must be bent, acquiring some properties of massive particles.

These conclusions, in essence, do not contradict experimental and theoretical data, obtained within the framework of standard model.

5.0. EM hadron models

According to SM there are two sorts of hadron: baryons, which contain three quarks, and mesons, which contains two quarks.

5.1. “Three quarks” model

We can suppose that in electromagnetic representation a barion is topologically the superposition of three knots and has the scheme of the trefoil knot (fig. 1):

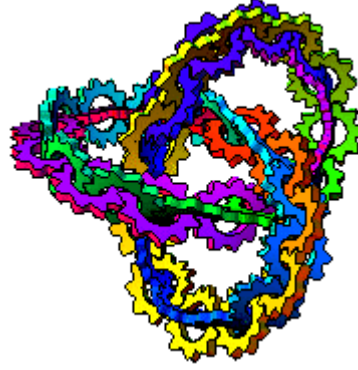


Fig.1

The figure 1 is taken from website (Möbius strip trefoil knot, MathWorld): <http://mathworld.wolfram.com/TrefoilKnot.html>, where the animation shows a series of gears motion along a Möbius strip trefoil knot as the electric and magnetic field vectors motion. A knot is defined as a closed, non-self-intersecting curve embedded in three dimensions. Knot theory was given its first impetus when Lord Kelvin proposed a theory that atoms of Democritus is vortex loops (Kelvin, 1867). The trefoil and its mirror image are not equivalent. In other words, the trefoil knot is chiral object. It is, however, invertible.

The equation of one loop (i.e. ring) is the Dirac equation that has a harmonic solution. Therefore, it can be supposed that the EM hadrons are the 3D superposition of two or three harmonic oscillations. On other words, the EM hadrons are similar to the space wave packets. According to Schreudinger (Schreudinger, 1926) (see also (Jammer, 1967), section 6.1) the wave packets, built from harmonic waves (oscillations), don't have a dispersion, i.e. they are stable. Thus, we can, as a first approximation, build the hadrons model as the space packet of the 3D superposition of three harmonics oscillations.

Here it must be noted that the superpositions of harmonic oscillations (i.e. Lissajous figures) are not the topological figures as knots, because they are the self-intersecting curves. But we can suppose that during the hadron formation as 3D Lissajous figures the loops will not intersect on account of the repulsion of currents.

The models were constructed by use of MathCAD-program. Probably the below models differ a lot from the real CWED particles and can not be used for calculation of the particle features. But they give some representation about them.

Thus, we suppose that the three-loops model (barion) is built from three harmonics oscillation. Let's choose the following oscillation parameters:

$$\omega_1 = 3, \quad \omega_2 = 2, \quad \omega_3 = 3, \quad ,$$

$$\phi_1 = \frac{\pi}{2}, \quad \phi_2 = \frac{\pi}{2}, \quad \phi_3 = 0,$$

$$r_1 = 2, \quad r_2 = 2, \quad r_3 = 2.$$

The argument has the view:

$$t_j := j \cdot 2 \cdot \frac{\pi}{N}, \text{ where } N := 200, j := 0..N, k := 0..N, v_k := k,$$

The harmonic oscillations are described by functions (for each co-ordinate axis):

$$X_{k,j} := r_1 \cdot \sin(\omega_1 \cdot t_j - \phi_1)$$

$$Y_{k,j} := r_2 \cdot \sin(\omega_2 \cdot t_j - \phi_2)$$

$$Z_{k,j} := r_3 \cdot \sin(\omega_3 \cdot t_j - \phi_3)$$

As result we obtain the following three-loops figure (fig.2):

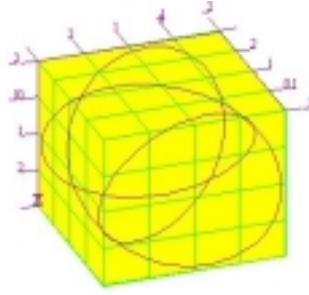


Fig. 2

To show the field plane twirling and twisting we change the parameter t_j to $t_j := \frac{j}{2.2}$. Then we have:

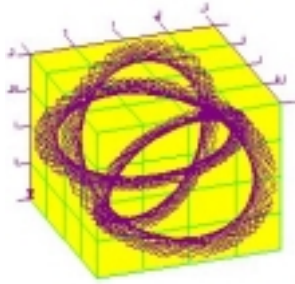


Fig. 3

5.2. "Two quarks" model

To build the two-loops model (meson) in the above proton model equations we choose: $\omega_1 = 1$ and $\phi_1 = 0$ and put $Z_{k,j} := 0$. Then we obtain the figure 4:

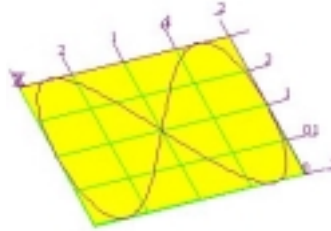


Fig. 4

It is necessary to note that depending on polarisation of the curvilinear photon (plain, circular, elliptic) the above models can have numerous different features.

We hope that the further investigations will allow us to build more realistic models, which will give us the opportunity to calculate the CWED particle features.

Discussion

The above electromagnetic representation of the Yang-Mills equations allows us to discuss some particularities of the QCD from point of view of CWED models:

1. The fractional charge of the quarks: according to the above results the electric field trajectory of the quarks not only has a curvature, but also a torsion; hence, the tangential current, generated by the electrical field vector transport, alternates along the space trajectory. Consequently, the electric charge of one knot, as an integral from this current, will be less, than the electron charge. But the total charge from all knots can be equal to the charge of the electron.

2. Quarks confinement: if quarks are two or three connected knots, quarks cannot exist in a free state.

3. The charges and masses of the quarks: in the CWED model quarks are defined by the rotation frequencies of each knot. From three-knot model it follows that figure 2 has a steady structure only at the certain circular frequencies ratio 3 : 3 : 2. So, for barion model two of quark charges and masses must be equal among them and not equal to mass of third quarks. Analogically, two-knot figure 4 have frequencies ratio 2 to 3 and therefore the meson model has two different quarks.

4. Non-linearity of the Yang-Mills equation: obviously, the Yang-Mills equation as the superposition of non-linear electromagnetic waves is the non-linear equation.

5. The gluons and photons analogy: according to the CWED topological models the gluons are the virtual photons, by which the knots interact between themselves.

6. The colours of quarks: it can be suppose that colours of quarks can identified with the quark currents since to each of the knots of the model have three different currents.

7. The colours of gluons: it can be suppose that colours of gluons can identified with two half-periods of virtual photons-gluons in respect that in the inner space of hadron these photons are bent and take the currents.

8. The strong interactions: possibly as it is supposed by Denis Wilkinson (Wofson College Lectures, 1980) the strong interaction can appear analogically to the Van der Waals forces in the atoms. As a result of the interpenetration of the atoms between them appears the forces, known by the name to Van der Waals, which presents the reflection of the specific side of electric force. Analogous correspondence occurs also for the force, which acts between the quarks and that caused by gluons, from one side, and by the force, which acts between the protons and the neutrons, with another. In this sense the strong interaction between the protons with respect to the strong interaction between the quarks corresponds to the appearance of Van der Waals force.

Of course the further analysis is needed to confirm or reject the above assertions, since they don't follow direct from the Yang-Mills equations. But as we see the CWED have the possibilities to explain many features of Standard Model.

Chapter 7. On elementary particles' spectra

1.0. Introduction

According to modern representations, all elementary particles are the bound states (including the excited states) of a small set of particles. For example, according to (Gottfried and Weisskopf, 1984): "The nucleon is simply a basic state of a compound spectrum of particles which we have named a baryon spectrum. Similarly pion is the lowest state of meson spectrum".

In present paper we wont show how in the framework of CWED the spectra of the particles as bound and excited states of a small set of some basic particles can be formed.

1.1. The spectra of characteristics of elementary particles

Generally each elementary particle is defined by a set of various characteristics: a mass, a spin, an electric charge, the strong and weak "charges" (i.e. the characteristics, which define intensity of strong and weak interaction), the numbers of "affinity" (numbers, owing to which one family of particles differs from another - lepton, baryon and other numbers), etc.

The particles, characterized by identical characteristics, except for any one of them, create a spectrum of elementary particles regarding this variable characteristic. For example, if as such variable characteristic the mass of particles is accepted, they speak about a mass spectrum of elementary particles.

According to the modern theory there are some limiting conditions of the composition of elementary particles, which can be named the conservation laws of this characteristic: e.g. the laws of conservation of energy, momentum, angular momentum, laws of conservation of an electric charge and charges of other interactions, laws of conservation of numbers of "affinity", etc. Some laws (principles) also exist, such as a principle of uncertainty of Heisenberg, which restrict the transition from one family or a spectrum of particles to another.

As is known, the existing field theory cannot explain the appearance of elementary particle characteristics and cannot deduce the majority of conservation laws of these characteristics: they are entered as consequences of experiments.

If to speak, for example, about mass spectra of particles, the following restrictions exist:

1) according to the energy-momentum conservation law the rest free light particles cannot break up to heavier particles, but heavy particles can break up to more light particles;

2) nevertheless, according to a uncertainty principle of Heisenberg, heavy particles cannot comprise the light particles as a ready particles (for example, the neutron cannot comprise electron as a free particle).

The conclusions of the quantum theory are undoubtedly correct and was confirmed by experiments, and we should show, that they do not contradict to the results of CWED.

2.0. A hypothesis of formation of spectra of elementary particles in CWED

Within the framework of CWED the electromagnetic twirled waves (EM-particles) possess the same characteristics, as quantum elementary particles. As we saw (see above chapters), the twirled harmonic waves, appearing here, can have integer or half spin, can be charged or neutral, etc. The mass of particles within the frameworks of CWED is the "stopped" energy of the twirled standing wave. Thus, roughly speaking, to a heavy particle by our representation corresponds the twirled wave of high frequency, and to light particle - the twirled wave of lower frequency. Thus, we should explain the existence of spectra of the particles relatively to all these particularities.

To the simple harmonic waves in Classical Electrodynamics (briefly CED), the twirled harmonic waves in CWED correspond. Does exist in CED the opportunity of coexistence of several waves as some material formation - an elementary particle, in which the characteristics of various waves can be superposed?

As we know, such opportunity actually exists and it consists in the waves superposition, which leads to various forms of coexistence of normal harmonic waves and to the appearance of complex non-harmonic waves, which "consist" from harmonic waves of various frequencies.

Analogically to the representations of classical theory of EM waves, whose non-linear generalization our theory is, we assume that *the reason of complication of EM particles and of appearance of its spectra is the superposition of simple (harmonic) twirled waves, and the reason of disintegrations of particles is the disintegration of the compound twirled waves.*

The purpose of our paper will be to show that such superposition exists and its description completely corresponds to modern theoretical representations and is in full accordance with the experimental data.

Since CWED is the non-linear generalization of classical (linear) electrodynamics, it is possible to assume, that the opportunity of the mathematical description of the waves spectra creation should exist already in CED. Besides, since mathematical description of CWED completely coincides with the mathematical description of quantum electrodynamics (QED), we should show that the similar forms exist in QED as well as in CWED.

2.1. Superposition of «linear» waves

Remember that under "linear" waves we understand the waves of the linear theory.

As it is known (Grawford, 1970), the any wave can be represented by superposition of more simple waves, named "modes" (terms: "simple harmonic oscillation", "harmonics", "normal oscillation", "own oscillation", "normal mode" or simply "mode" are identical). The properties of each mode of any compound system are very similar to properties of simple harmonic oscillator.

In many physical phenomena the system motion represents a superposition of two harmonic oscillations, having various angular frequencies ω_1 and ω_2 . These oscillations can, for example, correspond to two normal modes of the system, having two degrees of freedom. It is true as well for the quantum mechanical waves, described by quantum wave functions (see a known example of such system is the molecule of ammonia (Grawford, 1970)).

It is possible to illustrate this fact by the example of formation of an energy spectrum of electron in hydrogen atom. Really, the electron energy spectrum in an electron-proton system is from the general point of view a spectrum of electron masses. It is possible to speak about a basic mass (basic energy) in not excited state, and about a lot of masses of electron in the excited states, when electron receives additional portions of energy (mass). These portions are very small in comparison with the basic electron energy (mass), and we cannot consider the excited electrons as new particles. But, nevertheless, it does not exclude that these are the phenomenon of the same type as new particles' production. The increase of electron mass occurs due to absorption of photons, and the reduction of mass takes place due to emission of photons. On the other hand, we actually cannot tell here that the electron contains a photon as a ready particle.

It is easy to show (Grawford, 1970), that the change of electron energy as a result of its excitation by a photon corresponds to a hypothesis about the appearance of new particles owing to superposition of waves.

Let's consider the steady-states of the electron in one-dimensional potential well with infinitely high walls, whose coordinates are $z = -\frac{L}{2}$ and $z = +\frac{L}{2}$. We will also assume that the electron state is

defined by superposition of the basic state and the first excited state:

$$\psi(z, t) = \psi_1(z, t) + \psi_2(z, t), \quad (2.1)$$

where $\psi_1(z, t) = A_1 e^{-i\omega_1 t} \cos k_1 z$, $k_1 L = \pi$, $\psi_2(z, t) = A_2 e^{-i\omega_2 t} \sin k_2 z$, $k_2 L = 2\pi$

The probability of electron existence in the position z in the time moment t is equal to

$$\begin{aligned} |\psi(z, t)|^2 &= \left| A_1 e^{-i\omega_1 t} \cos k_1 z + A_2 e^{-i\omega_2 t} \sin k_2 z \right|^2 = \\ &= A_1^2 \cos^2 k_1 z + A_2^2 \sin^2 k_2 z + 2A_1 A_2 \cos k_1 z \cdot \sin k_2 z \cdot \cos(\omega_2 - \omega_1)t \end{aligned}, \quad (2.2)$$

We can see that the probability expression has a term, which makes harmonic oscillations with beats frequency between two Bohr frequencies ω_1 and ω_2 . The average electron position in space between the wells can be found the expression:

$$\bar{z} = \frac{\int z |\psi|^2 dz}{\int |\psi|^2 dz} = \frac{32L}{9\pi^2} \frac{A_1 A_2}{A_1^2 + A_2^2} \cos(\omega_2 - \omega_1)t, \quad (2.3)$$

where the integration is from one wall up to the other.

Obviously, the frequency of radiation is defined by beats frequency. Actually, electron is charged and, consequently, it will emit out the electromagnetic radiation of the same frequency, with which it oscillates. From the equation (1) we see, that average position of a charge oscillates with beats frequency $\omega_2 - \omega_1$. Therefore the frequency of radiation is equal to beats frequency between two stationary states:

$$\omega_{rad} = \omega_2 - \omega_1, \quad (1.4)$$

It is easy to understand that in the framework of CWED, the non-normalized quantum wave function is simply the wave field. As a consequence of this fact, the square of this wave function (i.e. the possibility density in the framework of QED) is the energy density.

As example of such problem in framework of CWED we will consider the calculation of more general case of the interference between waves of various frequencies. We will assume, that we have two EM waves 1 and 2, having electric fields \vec{E}_1 and \vec{E}_2 . The full field in the fixed point P of space will be the superposition of \vec{E}_1 and \vec{E}_2 . Using complex representation of oscillations, we will write the expression for superposition of oscillations:

$$\vec{E}(t) = E_1 e^{-i(\omega_1 t + \phi_1)} + E_2 e^{-i(\omega_2 t + \phi_2)}, \quad (2.5)$$

The energy flux is proportional to average value of $\vec{E}^2(t)$ for period T of the "fast" oscillations, appearing with average frequency:

$$\begin{aligned} 2 \langle E^2(T) \rangle &= \left| E_1 e^{-i(\omega_1 t + \phi_1)} + E_2 e^{-i(\omega_2 t + \phi_2)} \right|^2 = \\ &= E_1^2 + E_2^2 + 2E_1 E_2 \cdot \cos[(\omega_2 - \omega_1)t + (\phi_1 - \phi_2)] \end{aligned}, \quad (2.6)$$

As we see, the energy flux varies with relatively slow beats frequency $\omega_2 - \omega_1$.

3.0. Superposition of the twirled electromagnetic waves

Let's try to show here that at first the superposition of the twirled electromagnetic waves exists and secondly that owing to it, it is possible to receive all those results, which are known from the theory of the linear electromagnetic waves. In other words, it is necessary to show, that in this case there are actually spectra of particles, each of which represents complication of a basic twirled wave due to its superposition with other twirled waves.

As is known, all the phenomena of superposition of waves and their disintegration are described by Fourier theory (Fourier analysis-synthesis theory), in which it is shown, that any field can be synthesized from harmonic waves or analysed to harmonic waves. We will show that Fourier theory is true in case of the twirled waves as well as in case of linear waves.

3.1. The real and complex form solutions of the wave equation, as reflection of an objective reality

As is known, the wave equation

(CED form)	(CWED form)
------------	-------------

$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) \bar{\Phi}(y) = 0,$ <p>where</p> $\bar{\Phi}(y) = \{E_x, E_z, H_x, H_z\}$	$\left[(\hat{\alpha}_0 \hat{e})^2 - c^2 (\hat{\alpha} \hat{p})^2 \right] \Phi = 0,$ <p>where $\Phi = \begin{pmatrix} E_x \\ E_z \\ iH_x \\ iH_z \end{pmatrix},$</p> $\hat{e} = i\hbar \frac{\partial}{\partial t}, \hat{p} = -i\hbar \vec{\nabla} \text{ and } \hat{\alpha}_0; \hat{\alpha};$ $\hat{\beta} \equiv \hat{\alpha}_4 \text{ are Dirac's matrices}$
--	--

has the solution, which can be written down in the form of real periodic (in particular, trigonometric) functions, as well as in the form of complex (in particular, exponential) functions

$\bar{\Phi}(\vec{r}, t) = \bar{\Phi}_0 \cos(\omega t - \vec{k} \cdot \vec{r})$ $\bar{\Phi}(\vec{r}, t) = \bar{\Phi}'_0 \sin(\omega t - \vec{k} \cdot \vec{r})$	$\Phi = \Phi_0 e^{-i(\omega t \pm ky)}$ <p>or</p> $\begin{cases} \vec{E} = \vec{E}_0 e^{-i(\omega t \pm ky)}, \\ \vec{H} = \vec{H}_0 e^{-i(\omega t \pm ky)}, \end{cases}$
--	--

Nowadays it is considered that the representation of the solution of the wave equation (or oscillation equation) in complex form is only a formal mathematical method, since the final solutions should be real. It was also marked, that the use of complex representation is dictated only by the reasons of convenience, since in many cases the mathematical operations with exponential functions are easier, than with trigonometric.

We have shown (see the chapter 2), that within the framework of CWED the exponential solutions have an actual meaning, if we understand them in geometrical sense as the description of motion of a wave along a curvilinear (particularly the circular) trajectory. The equivalence of both descriptions becomes clear if we remember that the circular motion can represent as sum of two linear mutual-perpendicular oscillations. (We have noted that due to this fact the solutions of the wave equations of the quantum theory are not the real but complex wave functions).

Thus, it is possible to assume, that the existence of the real and complex solutions of the wave equation indicates the existence in the nature of two types of real objects: the linear and twirled (curvilinear) waves, so that the real functions describe the linear waves, and the complex functions describe the curvilinear (twirled) waves

As is known, the functions, which describe the complex periodic and non-periodic processes of non-harmonic type can be written by the sum of harmonic functions owing to Fourier analysis-synthesis theory. It must be noted that the Fourier analysis-synthesis theory allows to work equally both with real and complex functions.

From this the extremely important conclusion follows that all tools of the Fourier analysis-synthesis theory in complex representation is the mathematical apparatus, which describe the superposition and decomposition of complex twirled waves.

In other words, the complex representation of electromagnetic waves and all mathematical apparatus of the Fourier analysis-synthesis theory represent mathematical tool of CWED in the same degree as the mathematical apparatus of the real functions of Fourier analysis-synthesis theory represents the mathematical tool of usual linear Maxwell-Lorentz theory.

Due to above, the non-linear theory of the twirled waves is the theory in which the principle of superposition takes place as well as in the linear theory.

For this reason the linear Maxwell-Lorentz theory can be also written down in a complex form and it looks in such form simple and consistent. Transition from the twirled waves to linear (i.e. to one of components of the twirled wave) corresponds to transition from complex values to real .

Let us consider now some peculiarities of the Fourier analysis-synthesis theory in the case of superposition of the twirled waves.

4.0. Elementary particles as wave packets

As is known, in case of superposition of more than two classical linear harmonic waves the wave groups or wave packets are formed, which are limited in space.

In the quantum mechanics a wave packet (Physics Encyclopedia, V.1, 1960) is the concept, which denotes a matter waves' field, concentrated in the limited area. The probability to find a particle is differed from zero only in the area, occupied by a wave packet. It is possible to consider this wave field as result of superposition of the certain set of plane waves.

The possibility of composition and decomposition of plane waves is a simple result of a possibility to analyze any function in a Fourier series or Fourier integral.

It is meaningful to apply the concept of a wave packet when the wave numbers \vec{k} are grouped near to some \vec{k}_0 with small variation $\Delta\vec{k}$, $\Delta k \ll k_0$, since in this case the wave packet during significant time will move as a whole, with little deformation only and with the group speed $k_0 u = \left(\frac{d\omega}{dk} \right)_{k=k_0}$, corresponding to a speed of a particle, described by this wave packet. As is known, the smearing of the wave packet does not take place if it can be decomposed on standing waves, i.e. if in the decomposition series for each vector \vec{k} the vector $-\vec{k}$ with the same amplitude is also entered.

Since the superposition of linear waves leads to formation of the linear wave packets, it is logical to conclude that superposition of the twirled waves leads to formation of the twirled wave packets, i.e. to the compound electromagnetic elementary particles.

It is interesting that the representation of wave function by the Fourier series (in case of periodic function) or by the Fourier integral (in case of non-periodic function) contains the negative frequencies, which in the linear theory have no place:

<p>Real form:</p> $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin(\omega t))$ <p>where a_n, b_n are the Fourier coefficients.</p>	<p>Complex form:</p> $f(t) = \sum_{n=-\infty}^{\infty} c_n e^{-in\omega t}$ <p>, where c_n are the Fourier coefficients.</p>
---	---

As it is known (Matveev, 1985), in classical optics it take into consideration that $e^{i\omega t}$ describes here the complex unit vector, which rotates around the origin of coordinates in a positive direction (by a rule of the right screw). In the same time the complex unit vector $e^{-i\omega t}$ rotates in the negative direction. Thus, the appearance of the negative frequencies is connected with transition to the rotating complex vectors as to the basic functions of Fourier-transformation.

As a simple example of formation of a linear and non-linear wave packet, we will consider a packet formed by the equidistant rectangular frequency spectrum of waves of equal amplitudes. The description of superposition of such waves can be made in real (Grawford, 1970) as well as in a complex form (Matveev, 1985), that reflects the existence of the linear and non-linear world of particles.

We will find the exact expression for a packet $\psi(t)$ formed by superposition of N various harmonic components, which have equal amplitude A , an identical initial phase (equal to zero) and the frequencies distributed by regular intervals between the two frequencies: ω_1 and ω_2 . Generally we have:

<p>(real form)</p> $\psi(t) = A \cos \omega_1 t +$ $+ A \sum_{n=1}^{N-1} \cos(\omega_1 + n\delta\omega) t + A \cos \omega_2 t$	<p>(complex form)</p> $\psi(t) = A \sum_{n=0}^{N-1} e^{i(\omega_1 t + n\delta\omega t)}$
--	--

where $\delta\omega$ is the frequency difference of two next components, and $n = 1, 2, 3, \dots, N-1$ and $\omega_2 = \omega_1 + N\delta\omega$. This formula represents the complex wave function in the form of linear superposition of number of strictly harmonic components. It appears that this sum can be expressed in the form, which is the generalization of the case of two oscillations:

$$\psi(t) = A(t) \cos \omega_m t, \quad (4.1)$$

where $A(t) = A \frac{\sin(0,5N\delta\omega \cdot t)}{\sin(0,5\delta\omega \cdot t)}$ is the variable amplitude, $\omega_m = \frac{1}{2}(\omega_1 + \omega_2)$ is the average frequency of a wave packet. The amplitude $A(t)$ describes a wave packet envelope. It is possible to show (Grawford, 1970) that for a wave packet, Heisenberg uncertainty principle are true, what proves their wave origin. Apparently that in the case of twirled waves this principle described the particle size limit.

Since the twirled waves already are the space limited objects, it is possible to assume, that the electromagnetic particles should be combined not from infinite Fourier series, but they should be presented by the sum of the limited number of harmonics, i.e. of the twirled waves.

To describe the synthesis of the complex particles (packets) from more simple sub-packets, we will show, that any wave packet can be presented in the form of the sum of wave sub-packets. In this case, obviously, superposition (interaction) of several big packets can be considered not as superposition (interaction) of their separate harmonic components, but as superposition of their sub-packets (particles).

Let's consider the splitting of a big packet into two sub-packets. We will present a compound wave $\psi(t)$ (see above (4.1)) in the following form:

$$\begin{aligned} \psi(t) &= A \cos \omega_1 t + A \sum_{n=1}^{N-1} \cos(\omega_1 + n\delta\omega)t + A \cos \omega_2 t = \\ &= (A \cos \omega_1 t + A \sum_{m=1}^{N_1-1} \cos(\omega_1 + m\delta\omega)t + A \cos \omega_2' t) + \quad , \quad (4.2) \\ &+ (A \cos \omega_1' t + A \sum_{l=1}^{N_2-1} \cos(\omega_1' + l\delta\omega)t + A \cos \omega_2 t) \end{aligned}$$

where $N = N_1 + N_2$, $\omega_2' = \omega_1 + N_1\delta\omega$, $\omega_1' = \omega_1 + (N_1 + 1)\delta\omega = \omega_2' + \delta\omega$.

Thus, we can represent the wave packet $\psi(t)$ as two sub-packets:

$$\psi(t) = \psi_1(t) + \psi_2(t), \quad (4.3)$$

where $\psi_1(t) = A \cos \omega_1 t + A \sum_{m=1}^{N_1-1} \cos(\omega_1 + m\delta\omega)t + A \cos \omega_2' t$

$$\psi_2(t) = A \cos \omega_1' t + A \sum_{l=1}^{N_2-1} \cos(\omega_1' + l\delta\omega)t + A \cos \omega_2 t.$$

It is convenient to enter a shortening for a packet of waves $\sum \psi(t)$, where sigma means the sum of harmonic waves (in particular, a sub-packet). Then, representation of a packet in the form of the sum of sub-packets can be written down as:

$$\psi(t) = \sum \psi_1(t) + \dots + \sum \psi_2(t) = \sum_i \sum \psi_i(t), \quad (4.4)$$

From the above-stated calculations it is visible that decomposition on sub-packets (particles) is not unambiguous, since each one of the sub-packets can be grouped from harmonic waves in various ways. It is possible to assume, that the decay of the same particle on different channels can be considered as an opportunity of disintegration of packet on various sub-packets.

Using the above-stated reason it is easy to prove also that superposition (interaction) of sub-packets leads to the same consequences as interaction of separate harmonic waves, i.e. it leads to beats and to change of the energy level, independent from other non-interacting sub-packets.

Except for curvilinearity in CWED there is one more serious difference from linear electrodynamics: in CWED alongside with the full periodic twirled waves (bosons), exist also the half-period twirled waves (fermions). This creates a number of additional variants of the wave superposition, which are not present in linear electrodynamics. Besides, the curvilinearity enters into the physics one more characteristic of particles - the currents.

It is not difficult to understand also that the superposition of the twirled waves in comparison with the superposition of linear waves has more variants in a spatial arrangement of waves, and, hence, has more complex mathematical description. Actually we can see this in the case of description of hadrons (chapter 7).

It is easy to see, that the principle of superposition does not provide stability or, at least, metastability of compound electromagnetic particles. Thus, we should additionally find out the conditions of stability of the twirled waves.

5.0. The resonance theory of stability of elementary particles

As electromagnetic particles represent the spatial formations, here it is necessary to speak about spatial packets, which are formed by superposition of twirled waves of a various positions in space (e.g., by superposition of the twirled waves, which lie on three mutual-perpendicular coordinate planes).

As is known (Shpolskii, 1951), at the superposition of harmonic waves are formed the Lissajous figures of two various types. At commensurable frequencies of waves, the standing waves are formed; at incommensurable frequencies the motion of waves is referred to as quasi-periodic.

In the physics of waves and oscillations exist two sorts of the problems, leading to the appearance of the compound waves and oscillations.

An example of first type of problems is oscillation of the body volume (sphere, cylinder, torus, etc.), by which we can represent a particle. Here the suitable mechanical example is the oscillation of the sphere, prepared from a hydrophobic liquid and placed in water (for example, a sphere from mineral oil in water). In a microphysics the object, which possesses similar oscillations, is the drop model of a nucleus.

Problems concerning the same type are also the problems of oscillation of vortical rings in a perfect liquid or gas, studied by W. Kelvin (we will name conditionally such problems Kelvin's problems). In case of the oscillations of the linear vortex considered in work (Kelvin, 1867) he obtains the exact solution. Here Kelvin has compared the radiation spectra of the atoms, obtained little time before by Bunsen, to possible spectra of oscillation of vortex. Comparison of such type of oscillations with observable results is available e.g. in works (Paper collection, 1975) and (Kopiev and Chernyshev, 2000). (It is necessary to note, that in his articles W. Kelvin used the term "atoms" in sense of Democritus as the smallest indivisible constituents, i.e. in modern terminology as elementary particles).

Certain of the Kelvin significant conclusions from the paper "Atom as Vortex" we cite below:

"The author called attention to a very important property of the vortex atom. The dynamical theory of this subject require that the ultimate constitution of simple bodies should have one or more fundamental periods of vibration, as has a stringed instrument of one or more strings.

As the experiments illustrate, *the vortex atom has perfectly definite fundamental modes of vibration*, depending solely on that motion the existence of which constitutes it. The discovery of these fundamental modes forms an intensely interesting problem of pure mathematics. Even for a simple Helmholtz ring, the analytical difficulties, which it presents, are of a very formidable character. The author had attempted to work it for an infinitely long, straight, cylindrical vortex. For this case he was working out solutions corresponding to every possible description of infinitesimal vibration.

One very simple result, which he could now state is the following. Let such a vortex be given with its section differing from exact circular figure by an infinitesimal harmonic deviation of order i . This form will travel as waves round the axis of the cylinder in the same direction as the vortex rotation, with an angular velocity equal to $(i-1)/i$ of the angular velocity of this rotation. Hence, as the number of crests in a whole circumference is equal to i , for an harmonic deviation of order i there are $i-1$ periods of vibration in the period of revolution of the vortex. For the case $i=1$ there is no vibration, and the solution expresses merely an infinitesimally displaced vortex with its circular form unchanged. The case $i=2$ corresponds to elliptic deformation of the circular section; and for it the period of vibration is, therefore, simply the period of revolution. These results are, of course, applicable to the Helmholtz ring when the diameter of the approximately circular section is small in comparison with the diameter of the ring, as it is in the smoke-rings exhibited to the Society.

The lowest fundamental modes of the two forms of transverse vibrations of a ring, such as the vibrations that were seen in the experiments, must be much graver than the elliptic vibration of the section. It is probable that the vibrations which constitute the incandescence of sodium-vapour are analogous to those which the smoke-rings had exhibited".

As examples of other type of problems are oscillations of sound and electromagnetic waves into various types of the closed cavities (boxes), whose surface is motionless. Such cavities refer to as closed wave-guides or resonators and consequently we will conditionally name this type of problems the closed wave-guide or resonator problems. In the classical physics a set of researches is devoted to such type of problems. Examples of such type of problems are also eigenvalues problems of wave functions in the quantum mechanics, which we will consider briefly below.

The above first and second type of problems leads to solutions of type of the standing waves, which have the relative time stability.

Thus, it is possible to assume, that stability (or the relative stability named metastability) of electromagnetic particles is connected with a formation of standing waves.

As is known, a mathematical condition of appearance of standing waves is the proportionality of wavelength to the size of box (volume), in which the wave propagates. Therefore, at the study of a possible solution of these sorts of problems the basic role the limits play, which are imposed on propagation of waves or, in other words, the boundary states, imposed on wave functions.

Below we will show that from this boundary states follow the quantization conditions of characteristics of electromagnetic elementary particles.

5.1. Photon wave equation of classical electrodynamics

Let's consider again wave equation for the electric and magnetic field vectors (Matveev, 1989):

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \vec{\nabla}^2 \right) F(\vec{r}, t) = 0, \quad (5.1)$$

where \vec{F} is whichever of the EM wave functions.

The general harmonic solution of this wave equation has the complex $F(\vec{r}, t) = F(\vec{r})e^{-i\omega t} = F_0 e^{i(\vec{k}\vec{r} - \omega t)}$ or trigonometric forms $F(\vec{r}, t) = F_0 \cos(\vec{k}\vec{r} - \omega t)$, where

$\omega = 2\pi\nu$ is the angular frequency, ν is the linear frequency, $\vec{k} = \frac{2\pi}{\lambda} \frac{\vec{p}}{|\vec{p}|}$ is the wave vector,

$k = |\vec{k}|$ called the *wave number* so that $k = \omega/\nu = 2\pi/\nu T = 2\pi/\lambda$, T is the wave period and λ - wavelength. Note that we will obtain the same results whether we use the real forms or the complex.

Putting this solution in (5.1) we find for $F(\vec{r})$ the following equation for stationary waves:

$$(\vec{\nabla}^2 + k^2)F(\vec{r}) = 0, \quad (5.2)$$

Using these solutions it is also easy to obtain the dispersion law for EM waves:

$$\frac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2.$$

The equation (5.3) refers to as Helmholtz equation and is universal for the description of the coordinate dependence of harmonic waves' characteristics.

On the basis of this equation, was constructed the Kirchhoff diffraction and interference theory of light, which has excellently proved to be true an enormous experimental material, which can generalized in the case of the twirled waves' theory.

5.2. Wave equation solution for resonator

To analyse the electromagnetic wave equation solution for resonator we will take (Weinstein, 1957) an orthogonal box from metal with a , b and d sites as our model of resonator. We will show that this solution is the standing electromagnetic waves.

According to (5.2) the electric field must satisfy the equations

$$\begin{aligned} (\vec{\nabla}^2 + k^2)\vec{E}(\vec{r}) &= 0 \\ \vec{\nabla} \cdot \vec{E} &= 0 \end{aligned}$$

with the boundary state $\vec{E}_{||} = 0$ at the walls of the cavity (because inside the walls the electric energy will be rapidly dissipated by currents or polarization, the electric field intensity drops rapidly to zero into the walls). However, there could be an electric field *perpendicular* to the walls, because there could be the surface charge on the wall. This gives a possible solution:

$$\begin{cases} \vec{E}_x = E_{0x} k_x \cos k_x x \sin k_y y \sin k_z z \\ \vec{E}_y = E_{0y} k_y \sin k_x x \cos k_y y \sin k_z z, \\ \vec{E}_z = E_{0z} k_z \sin k_x x \sin k_y y \cos k_z z \end{cases} \quad (5.3)$$

For example, taking any x for which $\sin k_x x = 0$, the second and third terms above are identically zero, but the first term certainly isn't.

Also from $\text{div} \vec{E} = 0$ using (5.3) we find $\vec{\nabla} \vec{E} = (E_{0x} k_x + E_{0y} k_y + E_{0z} k_z) \sin k_x x \cdot \sin k_y y \cdot \sin k_z z = 0$ if choosing \vec{k} so that $\vec{k} \cdot \vec{E}_0 = 0$.

Here the wave equation requires $k_x = m\pi/a$, $k_y = n\pi/b$, $k_z = l\pi/d$, $\omega^2 = c^2(k_x^2 + k_y^2 + k_z^2)$ or $\omega = c\sqrt{k_x^2 + k_y^2 + k_z^2}$, where (l, m, n) are positive integers, e.g. (1, 1, 0) or (3, 2, 4), etc. In other words, each possible standing electromagnetic wave in the box corresponds to a point in the (k_x, k_y, k_z) space, labelled by three positive integers.

If we want also to obtain the general solution of the magnetic field, we first observe that the magnetic field satisfies the same equations and the boundary states as the electric field, and so the solution looks exactly the same as the electric solution. (An alternative way is to use $\vec{B} = \vec{\nabla} \times \vec{E}/i\omega$, which can be easily obtained from Maxwell theory).

Thus, the character of the general solution for EM wave in the cavity is the standing electromagnetic wave.

It is easy to see, that the stated above description of appearance of a resonance of the linear waves, if we make it in the complex form, will correspond to the appearance of the resonance of the curvilinear (twirled) waves.

Show now that the quantum wave equation solutions for the stationary states give the identical results.

6.0. The quantum wave equations and their solutions for stationary waves

6.1. De Broglie waves as twirled EM waves

De Broglie has assumed that material particles alongside with corpuscular properties have as well the wave properties so that to the energy and momentum of a particle in a corpuscular picture there correspond the wave frequency and wavelength in a wave picture. De Broglie has shown that in this case from relativistic transformations the parities strictly follow:

$$\varepsilon = \hbar\omega \quad \text{and} \quad \vec{p} = \frac{\hbar}{2\pi\lambda} \frac{\vec{p}}{|\vec{p}|} = \hbar\vec{k}$$

and the wave function of a material particle are described by the formulas:

$$\psi(\vec{r}, t) = \psi(\vec{r})e^{-i\omega t} = \psi_0 e^{i(\vec{k}\vec{r} - \omega t)} = \psi_0 e^{\frac{i}{\hbar}(\vec{p}\vec{r} - \varepsilon t)}$$

In the case of the de Broglie wave the dispersion law it is easy to find from the following energy-momentum conservation law for a particle:

$$\frac{\varepsilon^2}{c^2} = m_0^2 c^2 + \vec{p}^2$$

Really, replacing the energy and momentum by the wave characteristics, we will receive a dispersion correlation for waves of a matter:

$$\frac{\omega^2}{c^2} = \frac{m_0 c^2}{\hbar^2} + \vec{k}^2$$

We will show below that within the framework of CWED this dispersion correlation satisfies to the equation of the twirled semi-photon, which produce the Schrodinger or Dirac equations.

6.2. Helmholtz equation for de Broglie waves

The Helmholtz equation (5.2) describes the waves of various nature in homogeneous mediums and in vacuum with constant frequency ($\omega = \text{const}$). The constancy of wavelength is not supposed here.

Planck's correlation $\mathcal{E} = \hbar\omega$ shows that the condition $\omega = \text{const}$ entails the equality $\mathcal{E} = \text{const}$. Hence, Helmholtz equation can be applied to de Broglie waves at the description of motion of corpuscles in potential fields when their full energy is constant:

$$\mathcal{E} = \mathcal{E}_k + \mathcal{E}_p = p^2/2m + \mathcal{E}_p = \text{const}, \quad (6.1)$$

where $\mathcal{E}_k = p^2/2m$ is a kinetic energy, $\mathcal{E}_p(\vec{r}) \equiv V(\vec{r})$ is potential energy of a corpuscle in a field. From de Broglie correlation $\vec{p} = \hbar\vec{k}$ in view of (6.1) the equality follows:

$$k^2 = \frac{2m}{\hbar^2}(\mathcal{E} - \mathcal{E}_p), \quad (6.2)$$

Substituting the expression (6.2) for k^2 in (5.3) we receive the equation:

$$\left(\bar{\nabla}^2 + \frac{2m}{\hbar^2}(\mathcal{E} - \mathcal{E}_p) \right) F(\vec{r}) = 0, \quad (6.3),$$

named the Schroedinger stationary equation.

From this follows, that the existing calculation methods of the energy, momentum, angular momentum and other characteristics of particle state in the quantum field theory are calculations of resonance states of elementary particles in the various types of resonators, which in the quantum theory are usually named the potential wells. From the mathematical point of view these problems refer to as eigenvalues problems.

6.3. Quantization of state of the particle in the external field

The first calculations of quantum systems concerned the electron motion in the hydrogen atom. The formulas of quantization of electron characteristics in this case have been firstly found empirically (formulas of Balmer, Paschen, etc.) . Then, it has been shown that they turn out as consequence of conditions of Bohr quantization.

The generalization of Bohr quantization rules has been made independently by Wilson and Sommerfeld. They have shown, that in case of systems with any number of degree of freedom f it is possible to find such generalized coordinates q_1, q_2, \dots, q_f , in which the motion of system is separated on f harmonic oscillations; in this case a known rule of oscillator quantization can be applied for any of degrees of freedom. Owing to this generalization we receive f quantum conditions:

$$\oint p_1 dq_1 = \left(n_1 + \frac{1}{2} \right) \hbar, \quad \oint p_2 dq_2 = \left(n_2 + \frac{1}{2} \right) \hbar, \dots, \quad \oint p_f dq_f = \left(n_f + \frac{1}{2} \right) \hbar, \quad (7.1)$$

where the integers n_1, n_2, \dots, n_f refer to as quantum numbers.

As an example of the application of these rules it can be present the results of the hydrogen-like atom calculation (Shpolskii, 1951).

As de Broglie has shown, the Bohr or Wilson-Sommerfeld rules of quantisation define the conditions of the electron wavelengths integrality on various closed trajectories. Obviously, since any field can be represented as the oscillators sum, it is necessary to consider this rule as true for any quantum systems.

It is not difficult to see that within the framework of CWED these rules are natural rules of a resonance of the twirled electromagnetic waves, if we take into account a quantization rule of their energy according to Planck-de Broglie.

The results, received according to Wilson-Sommerfeld quantization rules, have later appeared as solutions of the wave equation for standing de Broglie waves (i.e. of the Schroedinger equation) for various sorts of potential wells (Shpolskii, 1951).

Thus, Schroedinger equation is the equation for calculation of resonance states of an electron wave in potential wells (resonators) of various type, boundary of wave motion in which are defined by potential energy of the system. Note, that the boundary states are expressed here by the same way, as in the classical theory of EM field:

$$\psi(a) = 0, \quad \psi(b) = 0, \quad \psi(d) = 0, \quad (7.5)$$

It is easy to show, that this problem is absolutely identical to the problem of stand EM wave in resonators (and also identical to the problem of oscillation of strings, membranes or elastic body). The

distinction is that the wave vector is not constant here, but by some complex way depends on spatial coordinates; or, in other words, the dispersion relation is here defined by the potential of system, which varies from a point to point according (6.2).

From above follows the conditions of formation of elementary particles' spectra.

7.0. Formation of elementary particles' spectra

According to our supposition the own spectra of elementary particles in CWED must arise in the same manner as the resonance states in any wave theory. The originality in comparison with calculation of stationary states of a particle in a field of other particles (solution for Schroedinger or Dirac electron equations) consists in the fact that *in this case we have not an external field (i.e. an external potential box), but the particles' themselves are like such a box.*

It is not difficult to imagine that medium in the electromagnetic resonator can possess a dispersion, depending on spatial coordinates under the same law, as potential energy in a potential well of quantum-mechanical problem. Recollecting that within the framework of CWED the EM wave function is identical to wave function of quantum mechanics, it is easy to understand that boundary states in a quantum-mechanical problem (7.5) must coincide with the boundary states in CED and CWED.

Unfortunately, the volume of the paper does not allow to consider other results of CWED and to give interpretation of many of SM results from the point of view of CWED. So, in the paper (Kyriakos, 2006) is developed the approach to calculation of the mass spectra of elementary particles within the framework of the resonance theory. Also in the paper (Kyriakos, 2004) was shown that in framework of CWED the description of the interaction is mathematically equivalent to the last in quantum field theory.

Bibliography

- Aharonov, Y. and Bohm, D. (1959). Significance of Electromagnetic Potentials in the Quantum Theory. *Phys. Rev. (Ser. 2)* **115**, 485-491.
- Akhiezer, A.I. and Berestetskii, W.B. (1965). *Quantum electrodynamics*. Moscow, Interscience publ., New York.
- Archibald, W.J. (1955). *Canadian Journal of Physics*, **33**, 565.
- Becker, R. (1933). *Theorie der Electricitaet, Band II, Electronentheorie*, Teubner Verlag, Leipzig und Berlin. New York
- Bethe, H. A. (1964) *Intermediate Quantum Mechanics*. W. A. Benjamin, Inc., New York - Amsterdam, Part II, Chapt. 17.
- Bialynicki-Birula, I. (1983). *Nonlinear Electrodynamics: Variations on a theme by Born and Infeld etc.*, World Scientific, Singapore.
- Bilenkii, S.M. (1982). *Introduction to the Physics of Electroweak Interactions*, Oxford, UK: Pergamon
- Bilenky, S.M., Giunti, C. and Kim, C.W. (2000) *Finally neutrino has mass*. Journal: *Int. J. Mod. Phys. A* **15**, 625 Preprint: KIAS-P99011, DFTT 08/99, hep-ph/9902462.
- Bo Thide, (2002). *Electromagnetic field theory*, Upsilon Books, Upsala, (<http://www.plasma.uu.se/CED/Book>)
- Born, M. (1953). *Proc. Phys. Soc.*, **66**, 402C, 501.
- Born, M. and Infeld, L. (1934). *Foundations of the new Field Theory*, *Proc. Roy. Soc. A* **144**, 425
- Brogie, L.de, (1923) *Comptes rendus*, Vol. 177, 1923, pp. 507-510
- Brogie, L.de. (1924). *Tentative theory of light quanta*. *Phil. Mag. and Journ. of Science*, v. XLVII, No. CCLXXVIII, 446.
- Brogie, de, L. (1925). *Recherches sur la theorie des quanta*. *Ann. d. Phys.*, v. III, dixieme serie, pp. 22-128.
- Brogie, L.de (1932a) *C.R.Acad.Sci. (Paris)* **195**, 536, 577, 862 ;
- Brogie, L.de (1932b) *C.R.Acad.Sci. (Paris)* **1973**, 1337
- Brogie, L.de (1934) *C.R.Acad.Sci. (Paris)* **198**, 135; etc
- Campolattoro, A.A. (1980). *International Journal of Theoretical Physics*, **19**, No 2, p.99-126.
- Cheng T.-P. and Li, L.-F. (1984). *Gauge Theory of Elementary Particle Physics* Clarendon Press, Oxford.
- Cheng, T.-P. and Li, L.-F. (2000) *Problems and Solution*, Clarendon Press, Oxford.

- Cook, R.J. (1982a). Photon dynamics. **A25**, 2164
- Cook, R.J. (1982b). Lorentz covariance of photon dynamics. **A26**, 2754
- Cronin, J. A., Greenberg, D.F., Telegdi, V. L. (1971). University of Chicago graduate problems in physics. Addison Wesley Publishing Co., Problem 6.7.
- Davydov, A. S., (1965). *Quantum Mechanics* (Pergamon, Oxford).
- Dirac, P. A. M., (1931). "Quantised singularities in the electromagnetic field".
Proc.Roy. Soc., **A33**, 60
- Dirac, P. A. M. (1931). *Proceeding Royal Society*, **A33**, 60.
- Dirac, P.A. M. (1938). Proc. Royal Soc. London **A167**, 148
- Ebert, H. (1957). *Physikalisches Taschenbuch*, F. Vieweg und Sohn, Braunschweig
- Ehrenfest, P. (1927). *Zs. F. Phys.*, **45**, 455
- Eisenhart, L. P. (1960). *A Treatise on the Differential Geometry of Curves and Surfaces*. New York: Dover.
- Euler, H. (1936). *Ann. D. Phys.* **26**, 398.
- Euler, H. Heisenberg W., (1936). *Zs. f. Phys.* **98**, 714
- Euler, H. Kockel B., (1935). *Naturwiss.*, **23**, 246
- Fermi, E. (1960). *Notes on quantum mechanics*. The University of Chicago press., (Section 34-4).
- Feynman, R. P (1965). Nobel lecture.
- Feynman, R.P. (1987). The reason for Antiparticles/Elementary particles and the laws of physics: the 1986 Dirac Memorial Lectures/. Cambridge University Press, 1987. – Pp. 1-59.
- Feynman, R.P., Leighton, R.B., Sands, M., (1963). *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA.), Vol. 3, Chap. 4, Sec 1.
- Feynman, R. P., Leighton, R. B., and Sands, (1989). *The Feynman Lectures on Physics, Vol. 2*.
- Fock, V. (1929). *J. Phys. Rad.*, 10, 392-405 (1929); *Z. Phys.*, 57 , 261-277;
- Fock, V., Ivanenko D., (1929). *C.R., Phys. Zs.*, 30, 648.
- Fock, V. (1929a). "Geometrisierung der Diracschen Theorie des Electrons", *Z. Phys.* 57 (1929) 261-277;
- Fock, V. (1929b) "L'Equation d'Onde de Dirac et la Geometrie de Riemann", *J. Phys. Rad.* 10 392-405;
- Fock, V. (1932). *The principles of the quantum mechanics*. Leningrad, (Part III, Chapt. I, § 5) (in Russian)
- Fock, V. and Ivanenko, D. (1929). *C.R., Phys. Zs.* 30, 648
- Frauenfelder, H. and Henley, E.M. (1974). *Subatomic physics*, Prentice-Hall, Inc., New Jersey.,
- Georgi, H. (1982). A unified theory of elementary particles and forces. - *Sci. Am.*, 244, No.4, P.40 (1981); *UFN*, т.136, vol.2, .
- Glashow, S.L. (1961). *Nucl. Phys.*, **22**, 597.
- Goenner, H. F. M. (2004). *Living Reviews Relativity*, 7,
(Online Article: <http://www.livingreviews.org/lrr-2004-2>)
- Gottfried, K. and Weisskopf, V.F. (1984). *Concepts of Particle Physics*. Oxford.
- Gould, Roy R. (1995). *Am. J. Phys.*, Vol 63, No. 2, February
- Grawford, F.S. (1970). *Waves*. Berkeley physics course.
- Gray, A. (1997). *Modern Differential Geometry of Curves and Surfaces with Mathematica*, 2nd ed. Boca Raton, FL: CRC Press, p. 186.
- Gsponer, A. and Hurni, J-P., (2005). Lanczos-Einstein-Petiau: From Dirac's equation to nonlinear wave mechanics (arXiv:physics/0508036 v2).
- Heisenberg, W. (1966). *Introduction to the unified field theory of elementary particles*. London.
- Inagaki, T. (1994). Quantum-mechanical approach to a free photon. *Phys.Rev.* **A49**, 2839.
- Infeld L. und van der Waerden B. L., (1933). *Berliner Berich.*, S. 380.
- Ivanenko, D. and Sokolov, A. (1949). *The classical field theory* (in Russian).
Moscow-Leningrad
- Jackson, J.D. (1999). *Classical Electrodynamics*, 3rd ed., 808 pp. Wiley, New York.
- Jammer, M (1967). *The conceptual development of quantum mechanics*. McGraw-Hill book Company,
- Kaempfer, F. A. (1965). *Concepts in quantum mechanics*, Academic Press, New York and London

- Kelvin, (Lord Kelvin). (1867). On Vortex Atoms. *Proc. Of the Royal Soc. Of Edinburgh*, Vol. VI, 1867, pp. 94-105; reprinted in *Phil. Mag. Vol. XXXIV*, 1867, pp. 15-24.
- Kockel, B. (1937). *Zs. f. Phys.* **107**, 153
- Koga, T. (1975). *International Journal of Theoretical Physics*, **13**, No 6, p.377-385.
- Konopinski, E.J. (1980). *Electromagnetic Fields and Relativistic Particles*, Chapter XIV, (McGraw-Hill, New York).
- Kopiev V. F., Chernyshev S.A. (2000). Oscillation of vortical ring etc., (In Russian). *UFN*, V. 170, No. 7.
- Korn, G.A. and Korn, Th. M., (1961). *Mathematical handbook for scientists and engineers*.
- Kyriakos, A.G. (2004a). <http://arXiv.org/abs/quant-ph/0407071>
- Kyriakos, A.G. (2004). <http://arxiv.org/abs/physics/0412095>
- Kyriakos, A.G. (2006). <http://arxiv.org/abs/hep-th/0604011>
- Lamb. H. (1931). *Lehrbuch der Hydrodynamik*, 2 Aufl. Leipzig, B.G.Treubner
- Landau, L.D. and Peierls, R. (1930). *Quantenelektrodynamik in Konfigurationsraum*. *Zs. F. Phys.*, **62**, 188.
- Larmor, J.J. (1900). *Aether and Matter*. Cambridge. Ch. 6.
- Lorentz, H.A. (1916). *The theory of electrons*. Leipzig. B.G. Teubner.
- Madelung, E. (1957). *Die mathematischen hilfsmittel des physikers*. Springer verlag, Berlin.
- Mandelstam, S., (1962). *Ann. Phys. New York*, **19**, 1
- Matveev, A.N. (1985). *Optics*. (In Russian). Moscow.
- Matveev, A.N. (1989). *Physics of Atom*. (In Russian). Moscow.
- Messiah, A. (1973). *Quantum mechanics*, Volum 2, North-Holland.
- Mie, G. (1912a). *Grundlagen einer Theorie der Materie*. – *Ann. der Physik*, **37**, 511.
- Mie, G. (1912b). *Grundlagen einer Theorie der Materie*. – *Ann. der Physik*, **39**, 1.
- Mie, G. (1913). *Grundlagen einer Theorie der Materie*. – *Ann. der Physik*, **40**, 1.
- Möbius strip, MathWorld: <http://mathworld.wolfram.com/MoebiusStrip.html>
- Möbius strip trefoil knot, MathWorld: <http://mathworld.wolfram.com/TrefoilKnot.html>
- Nambu, Y. and Jona-Lasinio, G., (1961) *Phys. Rev.*, **122**, No.1, 345-358.
- Nambu, Y. and Jona-Lasinio, G., (1961a) *Phys. Rev.*, **124**, 246.
- Naumov, A.I. (1984). *Physics of atomic nucleus and elementary particles* (in Russian). Moscow.
- Nishidzhima, K. (1982). “Week interactions at high energy”, *UFN*, т.136, vol.2m.
- Okun L. B., (1982). *Leptons and Quarks*, North Holland.
- Paper translation collection, (1959). *Non-linear quantum field theory*. Moscow (in Russian)
- Paper collection (1975) *Mechanics of vortical motion*. *News in the abroad physics*, (in Russian), Moscow.
- Pauli, W. (1958). *Theory of Relativity*. Pergamon, London.
- Pauli, W. and Weiskopf, V. (1934). *Helv. Acta*, 7, 709
- Peak, L. and Varvell, K., *The physics of the standard model*. Part 4, Lectures 4-15.
- Physics encyclopedia*. (1960). V.1, p.310, Wave packet. (In Russian). Moscow, 1960.
- Pich, A. (2000) *Aspects of quantum chromodynamics*. (arXiv: hep-ph/0001118 v.1.
- Pleban'ski, J. (1970). *Lecture Notes on Nonlinear Electrodynamics*, NORDITA, Copenhagen.
- Rodrigues, W.A., Jr. (2002). E-print arXiv: math-ph/0212034 v1.
- Ryder, L.H. (1985) *Quantum field theory*, 2nd ed., Cambridge Univ. Press., Cambridge, UK.
- Salam, A. (1969). *Proc. of the 8th Nobel Symposium on Elementary particle theory, relativistic groups and analicity*, edited by N. Svartholm,
- Schrodinger, E. (1926). *Naturwissenschaften*, Bd.14, S.664-666
- Schrodinger E. (1932). *Berliner Berich.*, S.105.
- Schrodinger, E. (1929); (1930); (1931a); (1931); *Sitzungber. Preuss. Akad. Wiss., (Berl.Ber.) (Phys.-math. kl.)*, S. 668-682 (1929); S. 418-428 (1930); S. 63-72 (1931); S. 238-347 (1931).
- Schwarzschild, R. (1903). *Goet. Nachr.math.-naturw. Kl.*, S.125.
- Schwinger, J. (1951). *Phys. Rev.*, **82**, 664.
- Schiff, L.T., (1955). *Quantum Mechanics*, 2nd edition, McGraw-Hill Book Company, Inc, New York.

- Scrinzi, A., Geissler, M. Brabec, Th. (2001). Phys. Rev. Lett. v. 86, p. 412.
- Shpolskii, E.W. (1951). Physics of atom. (In Russian). V.1, Moscow-Leningrad.
- Sokolov, A. and Ivanenko, D. (1952) *The quantum field theory* (in Russian), Moscow-Leningrad.
- Sommerfeld, A. (1958). A. Electrodynamics. Moscow.
- Ternov, A. I. (2000). *Electromagnetic features of massive neutrino*. SOJ, No. 10, (in Russian).
- Thomas, L.H. (1926). Nature, 117, p. 514
- Tomson, J.J. (1924). Electricity and matter. London
- Tonnelat M.-A., (1959). Les Principes de la Theorie Electromagnetique et de la Relativite. Masson et C., Paris.
- Trueba, J.L. and Ranada, A.F. (2000). Helicity in classical electrodynamics and its topological quantization. APEIRON, vol. 7, No. 1-2, January-April 2000.
- Weinstein, L.W. (1957). Electromagnetic waves. (In Russian). Moscow.
- Van der Waerden, B. L. (1929) „Spinoranalyse“, *Gott. Nachr.* 100-109.
- Weinberg, S. (1967). Phys. Rev. Lett., **19**, 1264.
- Weisskopf, V. (1936). Kgl. Danske Vidensk. Selskab., Mat.-fys. Medd., **14**, 6, 1.
- Wheeler, J.A., and Feynman, R.P. (1945). Reviews of Modern Physics, 17, 157.
- Wheeler, J.A. (1957). Reviews of Modern Physics 29, 463.
- Wofson College Lectures (1980). The nature of matter. Edited by J.H. Mulvey. Clarendon Press, Oxford, 1981